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# Affine Gap Alignment

— CS181 Fall 2021 —

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# Definitions: Inputs and Outputs

Inputs:  $\langle X, Y, \alpha, \mu, \gamma, \tau \rangle$

- $X, Y$  = strings of length  $m, n$  with characters indexed by  $i, j$ , respectively
- $\alpha$  = match score
- $\mu$  = mismatch penalty
- $\gamma$  = gap opening penalty
- $\tau$  = gap extension penalty (single-letter gap penalty)

Output: An alignment which maximizes the following score:

$$\alpha(\# \text{ matches}) - \mu(\# \text{ mismatches}) - \gamma(\# \text{ gap clusters}) - \tau(\# \text{ single-letter gaps})$$

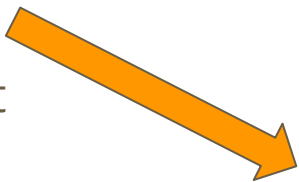
# Definitions: Auxiliary Data Structures

Matrices:  $V$ ,  $G$ ,  $E$ ,  $F$

- $V$  = the best-score matrix
- $G$  = the match-mismatch matrix
- $E$  = the  $X$ -gap matrix
- $F$  = the  $Y$ -gap matrix

# The Algorithm:

- 1) Initialize the matrices
- 2) Apply the recurrence relations to fill each matrix
- 3) Traceback through  $V$  (not shown)



```
V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end
```

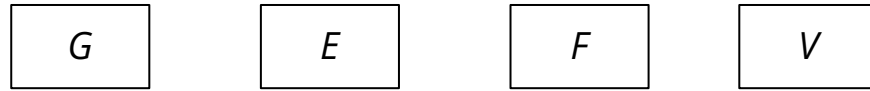
# An Example:

- $X = \text{ATCGGC}$
- $Y = \text{AGC}$
- $\alpha = 2$  (score = +2)
- $\mu = 1$  (penalty = -1)
- $\gamma = 2$  (penalty = -2)
- $\tau = 1$  (penalty = -1)

```
V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ←  $\begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$ 
  E(i,j) ←  $\max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$ 
  F(i,j) ←  $\max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$ 
  V(i,j) ←  $\max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$ 
end
```

# Tips for Initialization

1. Update the scores with the right recurrence relation math.
2. Update the backpointers to know where the score originated from.
3. The  $i$  represents the column and the  $j$  represents a row number in our example.



<b>G</b>		<b>A</b>	<b>T</b>	<b>C</b>	<b>G</b>	<b>G</b>	<b>C</b>
	-	-	-	-	-	-	-
<b>A</b>	-						
<b>G</b>	-						
<b>C</b>	-						

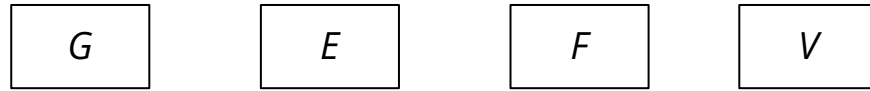
\*0th row, 0th column in  $G$  is unused  $\rightarrow$  initialize with error values

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



E	A	T	C	G	G	C
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	-3					
G	-4					
C	-5					

\*0th row in  $E$  is unspecified  $\rightarrow$  initialize with negative infinity to favor opening gaps from  $V$  once we start calculating down

$$V(0,0) \leftarrow 0$$

```
for j ← 1 to n do
  | V(0, j) = E(0, j) ← -γ - jτ
end
```

```
for i ← 1 to m do
  | V(i, 0) = F(i, 0) ← -γ - iτ
end
```

```
for i ← 1 to m, j ← 1 to n do
```

$$G(i, j) \leftarrow \begin{cases} V(i-1, j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1, j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i, j) \leftarrow \max \begin{cases} E(i, j-1) - \tau \\ V(i, j-1) - \gamma - \tau \end{cases}$$

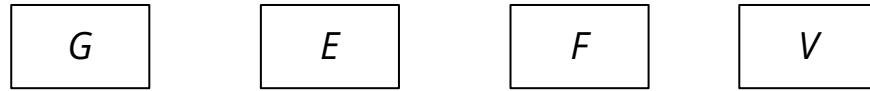
$$F(i, j) \leftarrow \max \begin{cases} F(i-1, j) - \tau \\ V(i-1, j) - \gamma - \tau \end{cases}$$

$$V(i, j) \leftarrow \max \begin{cases} G(i, j) \\ E(i, j) \\ F(i, j) \end{cases}$$

```
end
```

$$\begin{matrix} \alpha = 2 & \gamma = 2 \\ \mu = 1 & \tau = 1 \end{matrix}$$





E		A	T	C	G	G	C
	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$						
G	$-4_E$						
C	$-5_E$						

**\*Also keep backpointers to let you know which matrix was used to compute the score in each cell**

$V(0,0) \leftarrow 0$

**for**  $j \leftarrow 1$  **to**  $n$  **do**  
 |  $V(0,j) = E(0,j) \leftarrow -\gamma - j\tau$   
**end**

**for**  $i \leftarrow 1$  **to**  $m$  **do**  
 |  $V(i,0) = F(i,0) \leftarrow -\gamma - i\tau$   
**end**

**for**  $i \leftarrow 1$  **to**  $m, j \leftarrow 1$  **to**  $n$  **do**

$G(i,j) \leftarrow \begin{cases} V(i-1, j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1, j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$

$E(i,j) \leftarrow \max \begin{cases} E(i, j-1) - \tau \\ V(i, j-1) - \gamma - \tau \end{cases}$

$F(i,j) \leftarrow \max \begin{cases} F(i-1, j) - \tau \\ V(i-1, j) - \gamma - \tau \end{cases}$

$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$

**end**

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



F		A	T	C	G	G	C
	$-\infty$	-3	-4	-5	-6	-7	-8
A	$-\infty$						
G	$-\infty$						
C	$-\infty$						

\*0th column in  $F$  is unspecified  $\rightarrow$  initialize with negative infinity to favor opening gaps from  $V$  once we start calculating across

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



F		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$						
G	$-\infty$						
C	$-\infty$						

**\*Also keep backpointers to let you know which matrix was used to compute the score in each cell**

```

V(0,0) ← 0
for j ← 1 to n do
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                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
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V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$						
G	$-4_E$						
C	$-5_E$						

\*In V, keep track of which matrix gave you each score

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end

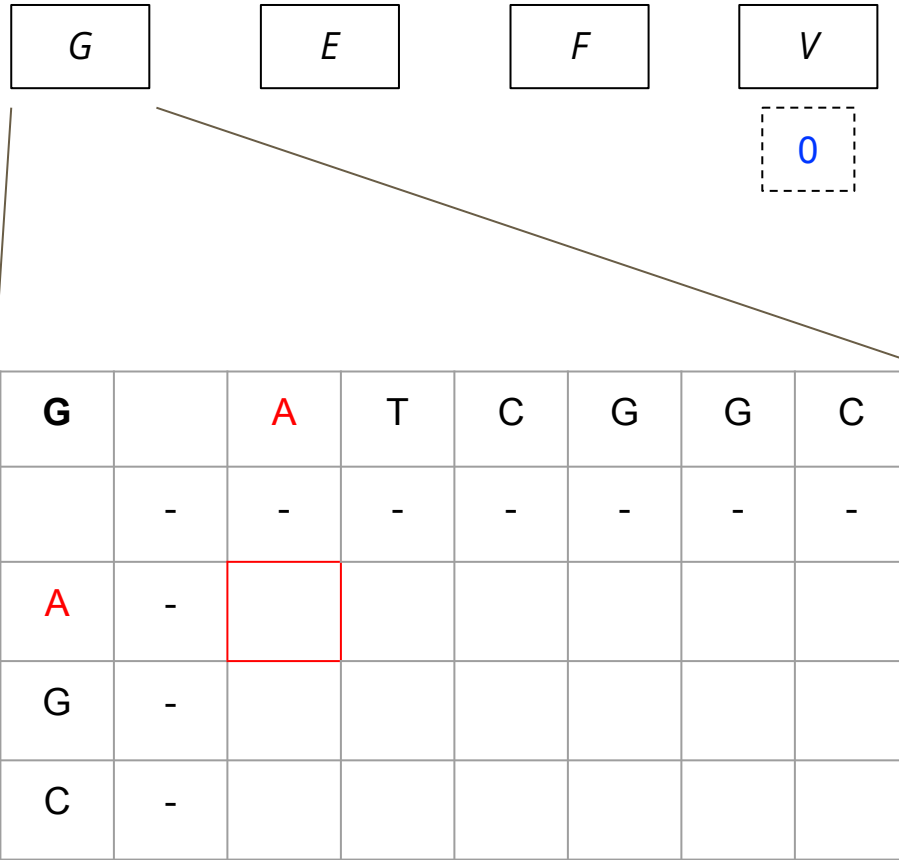
```

```

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  E(i,j) ← max { E(i,j-1) - τ
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                F(i,j)
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```

$\alpha = 2$	$\gamma = 2$
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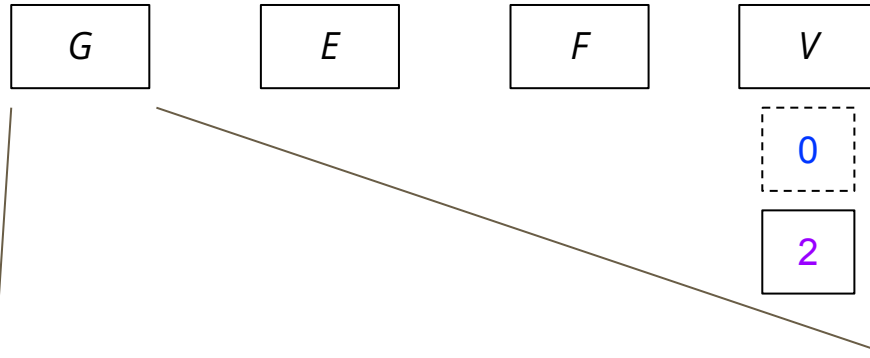


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            V(i,j) ← max { G(i,j)
                          E(i,j)
                          F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
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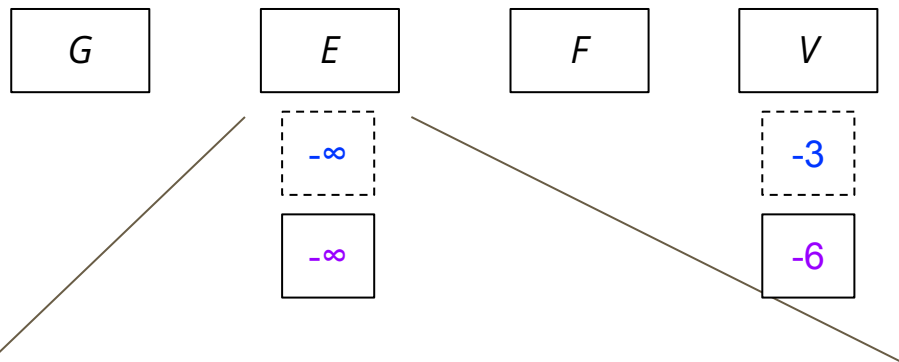
G		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2					
G	-						
C	-						

\*We don't need backpointers in G because all entries are computed from the same cell in V

```

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            E(i,j) ← max { E(i,j-1) - τ
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                          V(i-1,j) - γ - τ
            V(i,j) ← max { G(i,j)
                          E(i,j)
                          F(i,j)
  end
  
```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



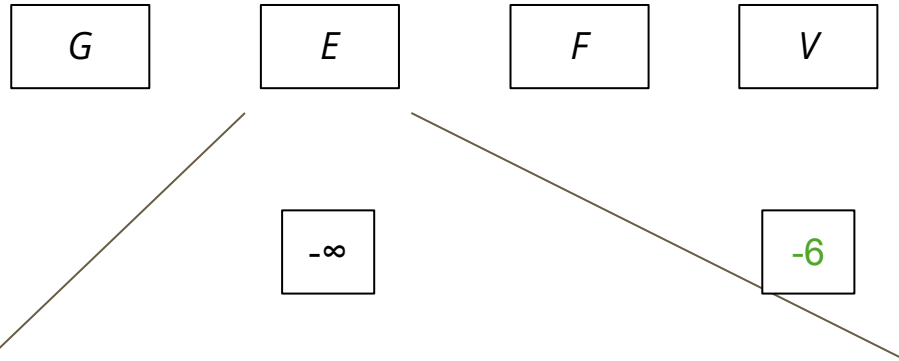
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                F(i,j)
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$\alpha = 2$	$\gamma = 2$
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E	A	T	C	G	G	C
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$				
G	$-4_E$					
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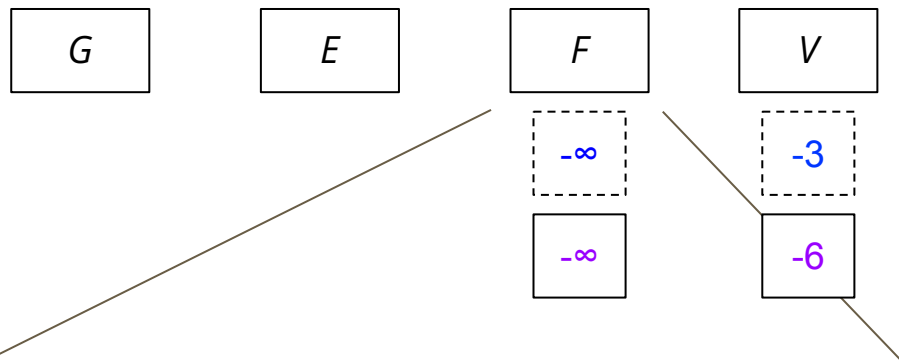
```

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for i ← 1 to m, j ← 1 to n do
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                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$





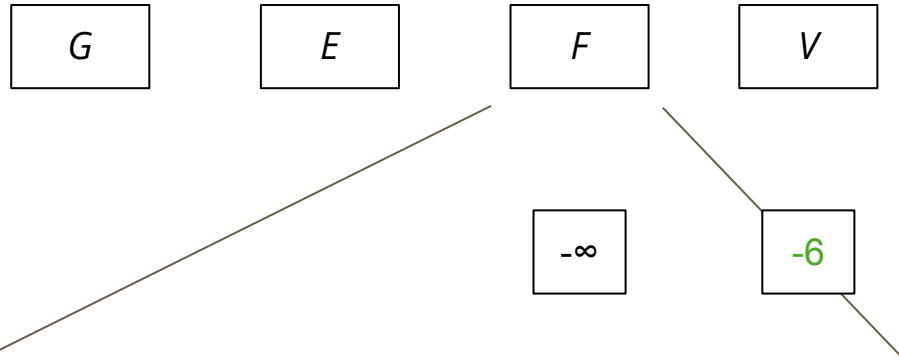
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$					
G	$-\infty$					
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                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$   $\gamma = 2$   
 $\mu = 1$   $\tau = 1$



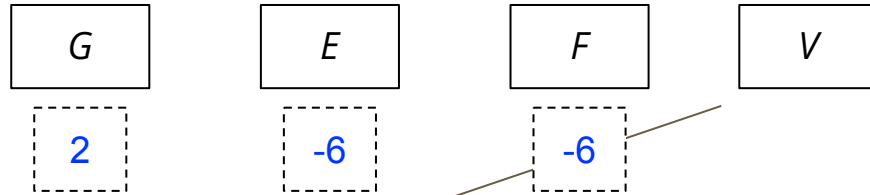
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$				
G	$-\infty$					
C	$-\infty$					

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  V(i,j) ← max { G(i,j)
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end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



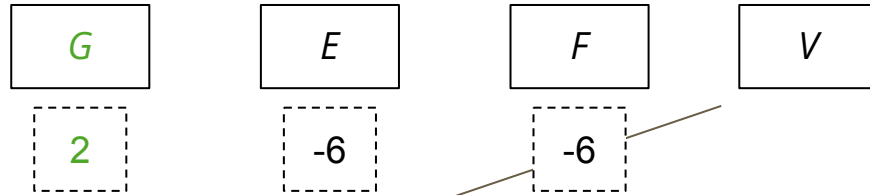
V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$						
G	$-4_E$						
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end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



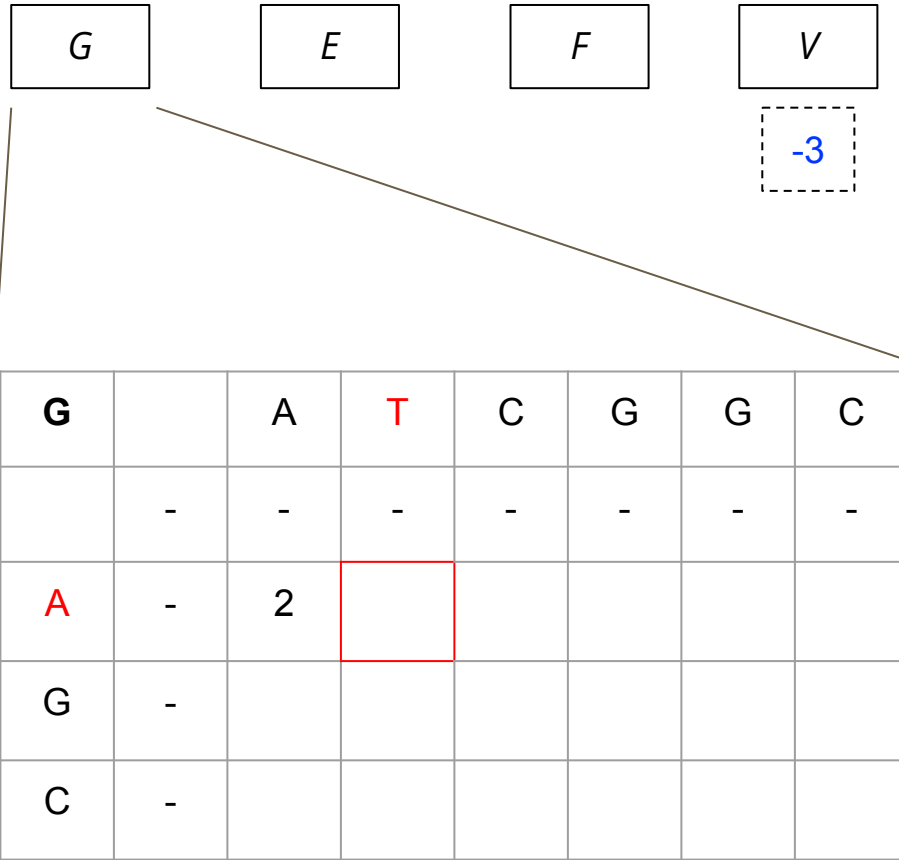
V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$					
G	$-4_E$						
C	$-5_E$						

```

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  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$

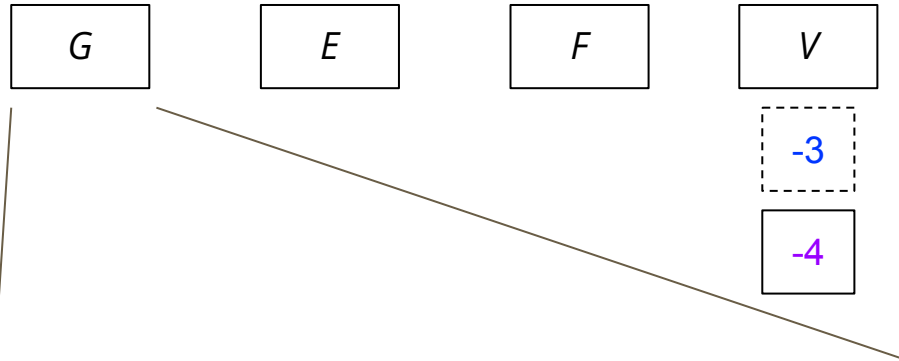


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end
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end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← {
    V(i-1,j-1) + α if x_i = y_j
    V(i-1,j-1) - μ if x_i ≠ y_j
  }
  E(i,j) ← max {
    E(i,j-1) - τ
    V(i,j-1) - γ - τ
  }
  F(i,j) ← max {
    F(i-1,j) - τ
    V(i-1,j) - γ - τ
  }
  V(i,j) ← max {
    G(i,j)
    E(i,j)
    F(i,j)
  }
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



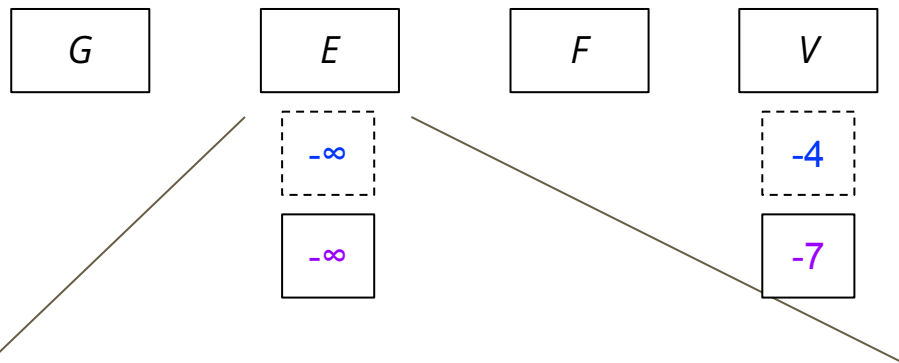
G		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2	-4				
G	-						
C	-						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            | V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                | V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                | V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                | E(i,j)
                | F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



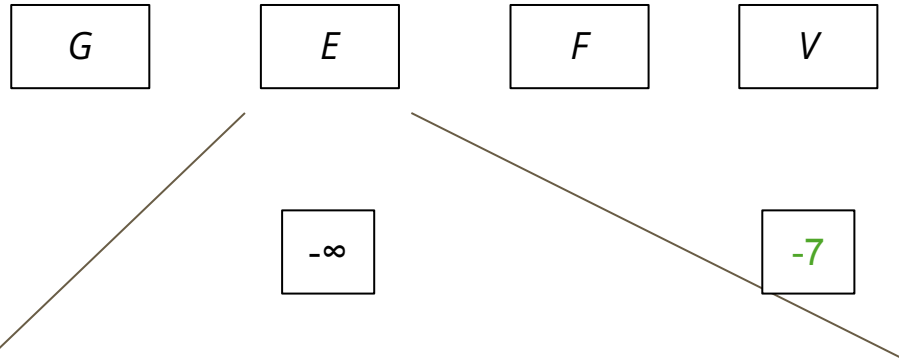
E	A	T	C	G	G	C
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$				
G	$-4_E$					
C	$-5_E$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$   $\gamma = 2$   
 $\mu = 1$   $\tau = 1$



E	A	T	C	G	G	C
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$	$-7_V$			
G	$-4_E$					
C	$-5_E$					

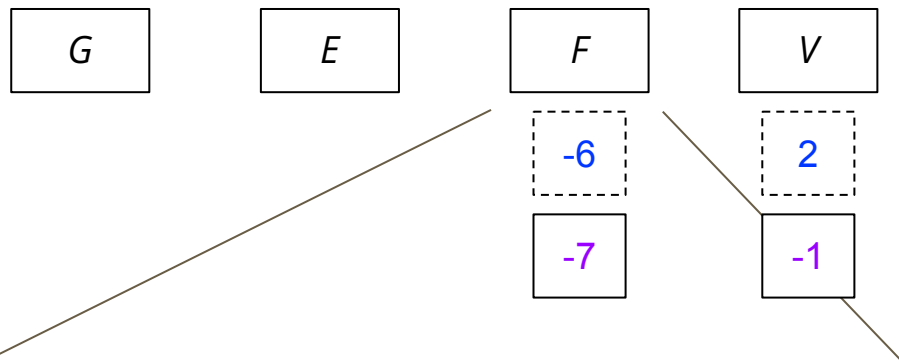
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
             V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                 V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                 V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                 E(i,j)
                 F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$





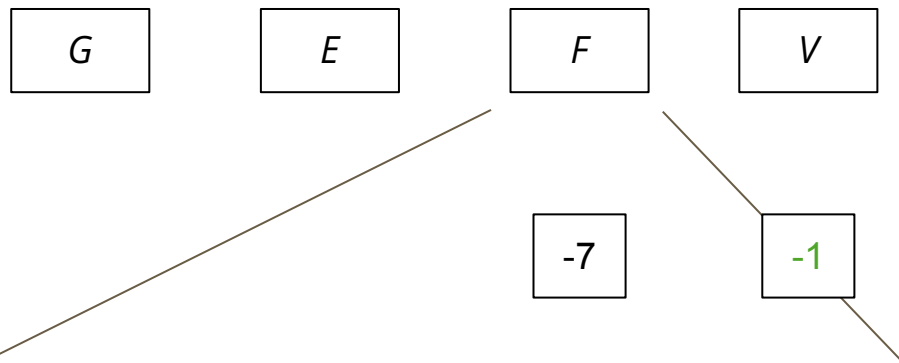
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-6_V$					
G	$-\infty$					
C	$-\infty$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$   $\gamma = 2$   
 $\mu = 1$   $\tau = 1$



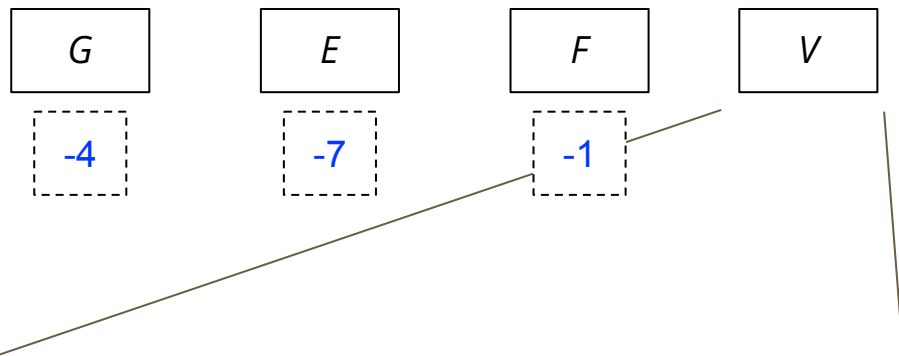
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-1_V$				
G	$-\infty$					
C	$-\infty$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



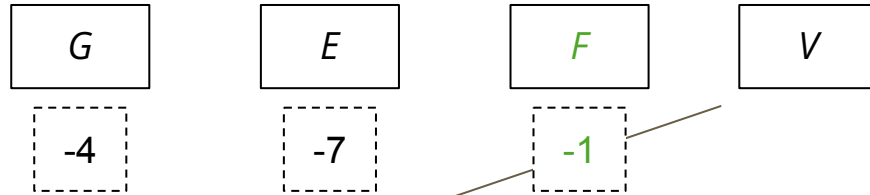
V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$					
G	$-4_E$						
C	$-5_E$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



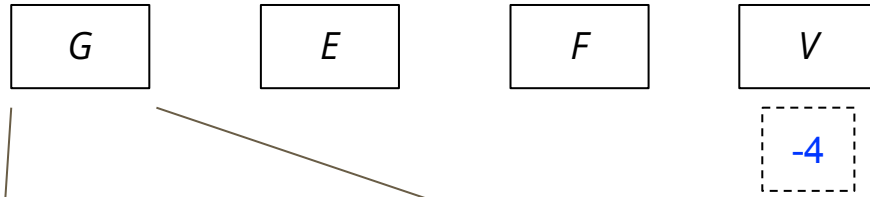
V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$				
G	$-4_E$						
C	$-5_E$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



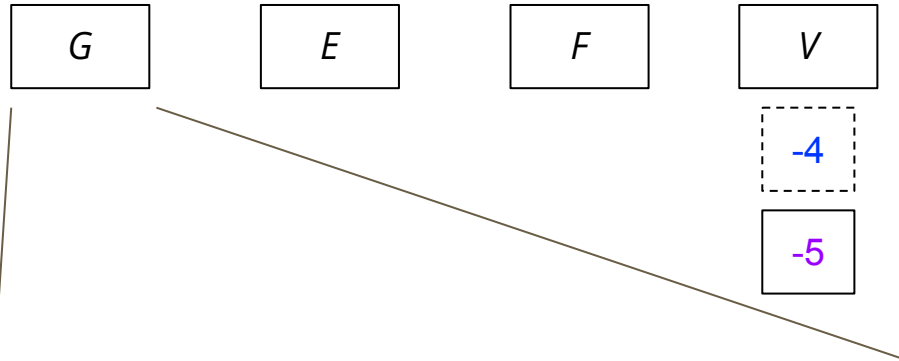
G		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2	-4				
G	-						
C	-						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← {
    V(i-1,j-1) + α if xi = yj
    V(i-1,j-1) - μ if xi ≠ yj
  }
  E(i,j) ← max {
    E(i,j-1) - τ
    V(i,j-1) - γ - τ
  }
  F(i,j) ← max {
    F(i-1,j) - τ
    V(i-1,j) - γ - τ
  }
  V(i,j) ← max {
    G(i,j)
    E(i,j)
    F(i,j)
  }
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



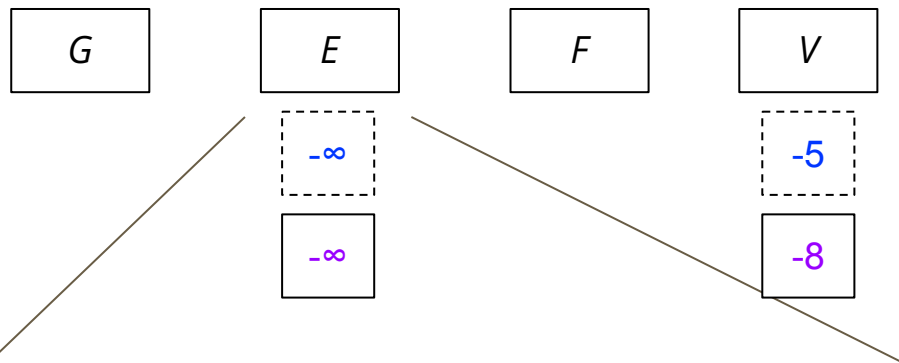
G		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2	-4	-5			
G	-						
C	-						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            | V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                | V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                | V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                | E(i,j)
                | F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



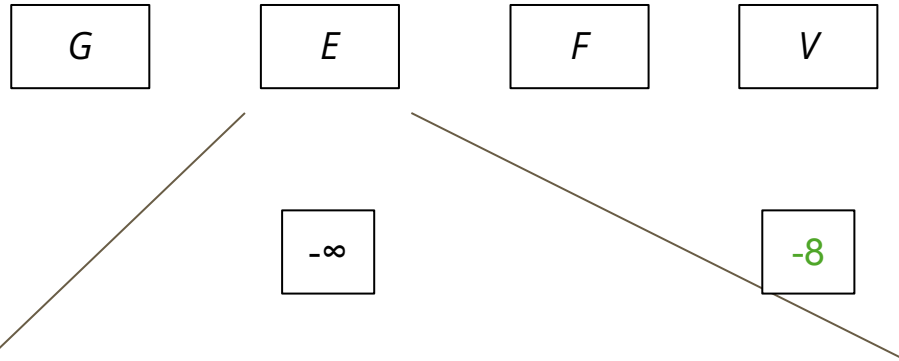
E	A	T	C	G	G	C
-∞	-∞	-∞	-∞	-∞	-∞	-∞
A	-3 <sub>V</sub>	-6 <sub>V</sub>	-7 <sub>V</sub>			
G	-4 <sub>E</sub>					
C	-5 <sub>E</sub>					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



E	A	T	C	G	G	C
-∞	-∞	-∞	-∞	-∞	-∞	-∞
A	-3 <sub>V</sub>	-6 <sub>V</sub>	-7 <sub>V</sub>	-8 <sub>V</sub>		
G	-4 <sub>E</sub>					
C	-5 <sub>E</sub>					

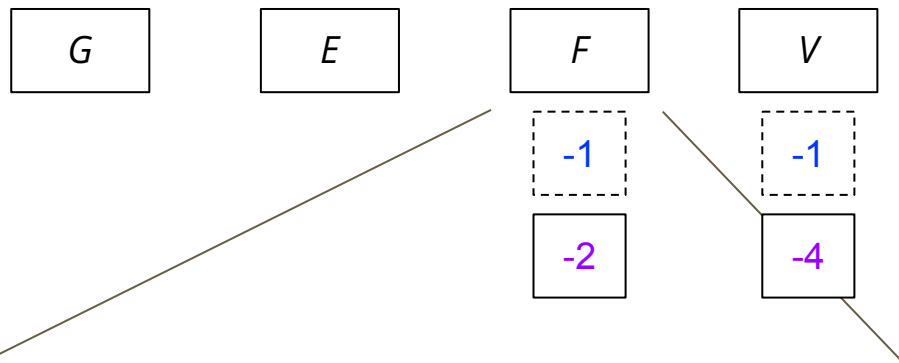
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$





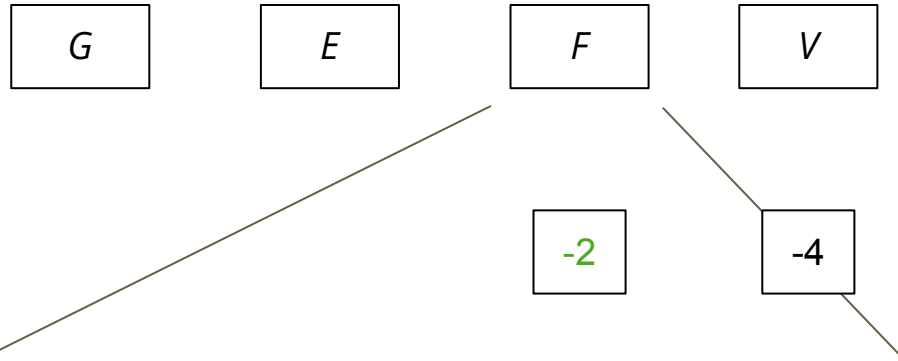
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-1_V$				
G	$-\infty$					
C	$-\infty$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$   $\gamma = 2$   
 $\mu = 1$   $\tau = 1$



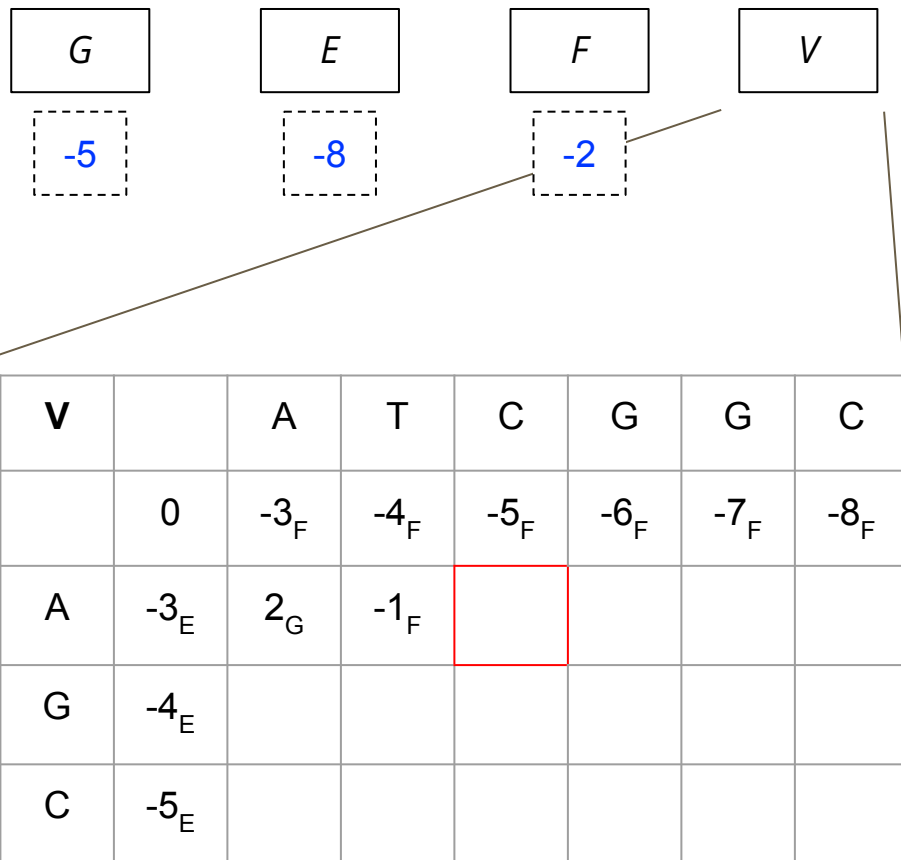
F		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$			
G	$-\infty$						
C	$-\infty$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$

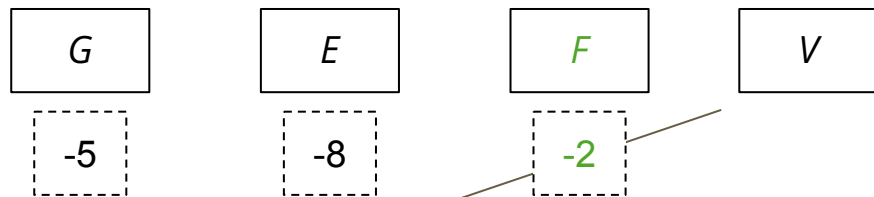


```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if xi = yj
            V(i-1,j-1) - μ if xi ≠ yj
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$			
G	$-4_E$						
C	$-5_E$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

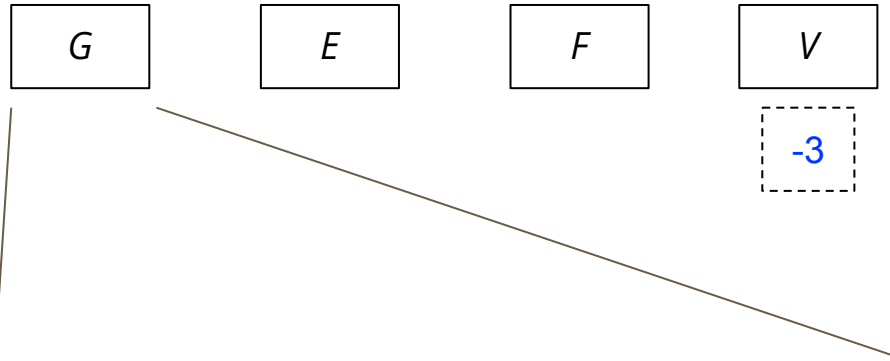
$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$

## And so on...

The algorithm continues like so until all the matrices are filled.

We'll skip ahead, stopping at some interesting intermediate states which cover new branches in the algorithm.

As an exercise, try filling out these matrices on your own and checking the values against our final solution!



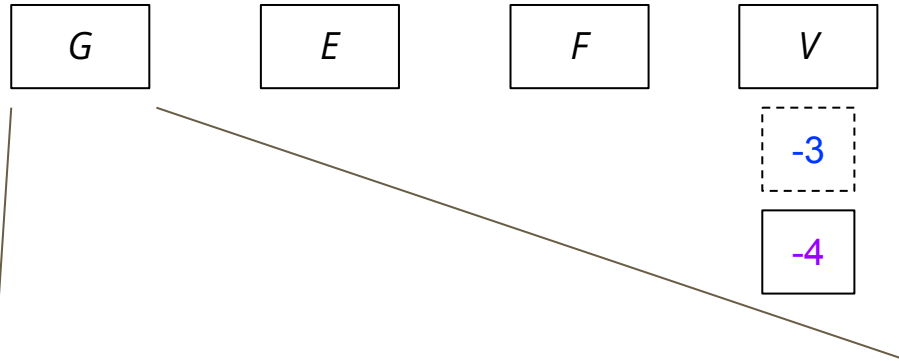
<b>G</b>		<b>A</b>	<b>T</b>	<b>C</b>	<b>G</b>	<b>G</b>	<b>C</b>
	-	-	-	-	-	-	-
<b>A</b>	-	2	-4	-5	-6	-7	-8
<b>G</b>	-						
<b>C</b>	-						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← {
    V(i-1,j-1) + α if x_i = y_j
    V(i-1,j-1) - μ if x_i ≠ y_j
  }
  E(i,j) ← max {
    E(i,j-1) - τ
    V(i,j-1) - γ - τ
  }
  F(i,j) ← max {
    F(i-1,j) - τ
    V(i-1,j) - γ - τ
  }
  V(i,j) ← max {
    G(i,j)
    E(i,j)
    F(i,j)
  }
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



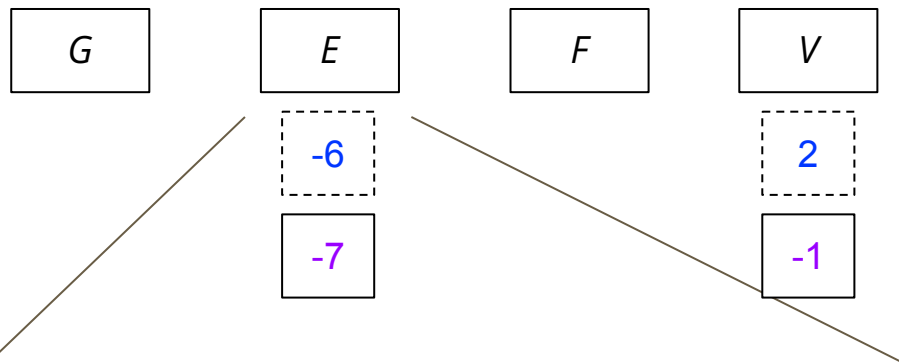
<b>G</b>		<b>A</b>	<b>T</b>	<b>C</b>	<b>G</b>	<b>G</b>	<b>C</b>
	-	-	-	-	-	-	-
<b>A</b>	-	2	-4	-5	-6	-7	-8
<b>G</b>	-	-4					
<b>C</b>	-						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            { V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                { V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                { V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                { E(i,j)
                { F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



E		A	T	C	G	G	C
	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$	$-7_V$	$-8_V$	$-9_V$	$-10_V$	$-11_V$
G	$-4_E$						
C	$-5_E$						

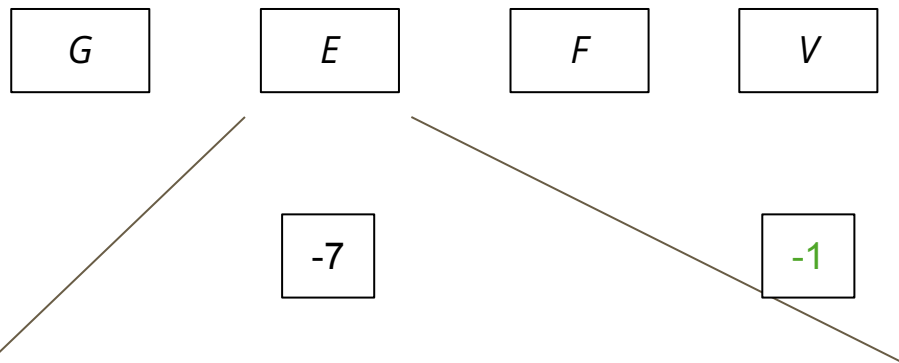
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
             V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                 V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                 V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                 E(i,j)
                 F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$





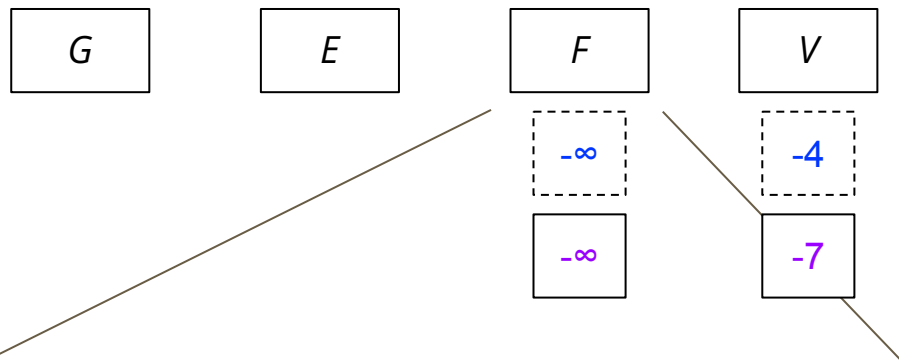
E		A	T	C	G	G	C
	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$	$-7_V$	$-8_V$	$-9_V$	$-10_V$	$-11_V$
G	$-4_E$	$-1_V$					
C	$-5_E$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



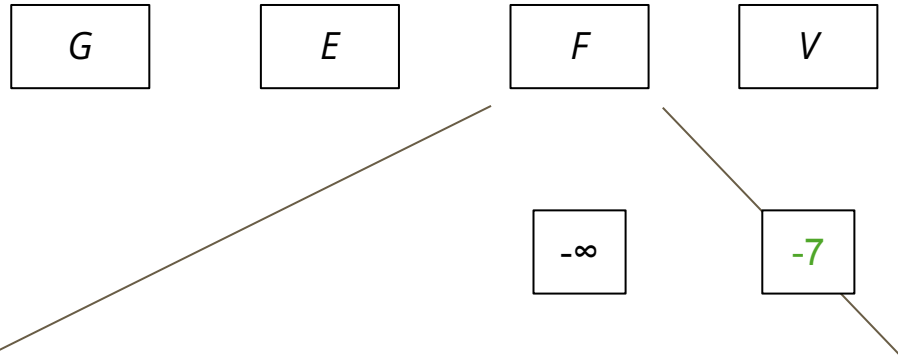
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$
G	$-\infty$					
C	$-\infty$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



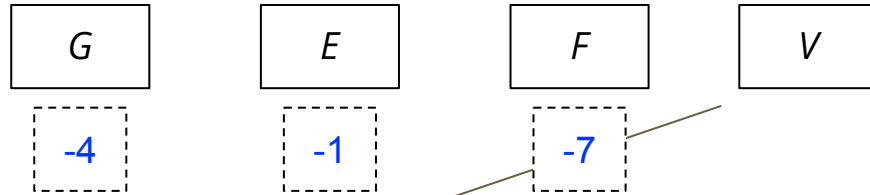
F		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$					
C	$-\infty$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



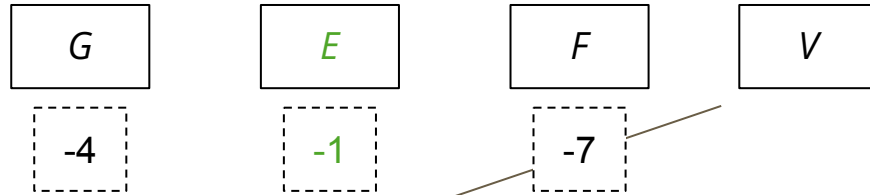
V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$						
C	$-5_E$						

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$	$-1_E$					
C	$-5_E$						

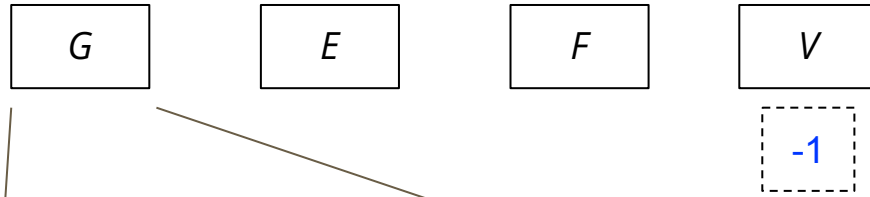
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$





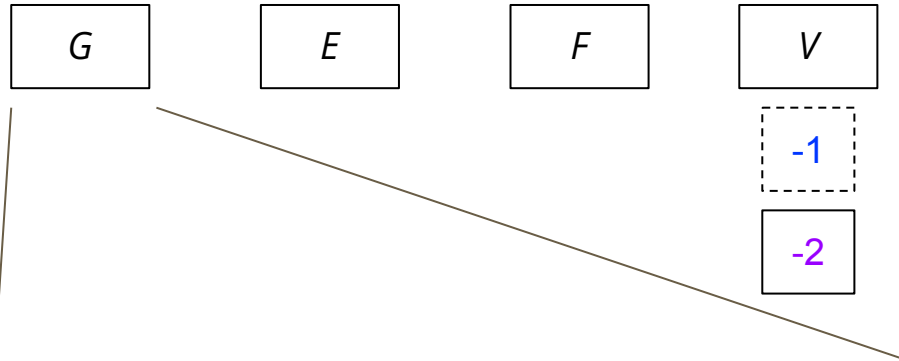
<b>G</b>		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2	-4	-5	-6	-7	-8
G	-	-4	1	-2	0	-1	-5
C	-	-5					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
            E(i,j) ← max { E(i,j-1) - τ
                          V(i,j-1) - γ - τ
            F(i,j) ← max { F(i-1,j) - τ
                          V(i-1,j) - γ - τ
            V(i,j) ← max { G(i,j)
                          E(i,j)
                          F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



<b>G</b>		A	T	C	G	G	C
	-	-	-	-	-	-	-
A	-	2	-4	-5	-6	-7	-8
G	-	-4	1	-2	0	-1	-5
C	-	-5	-2				

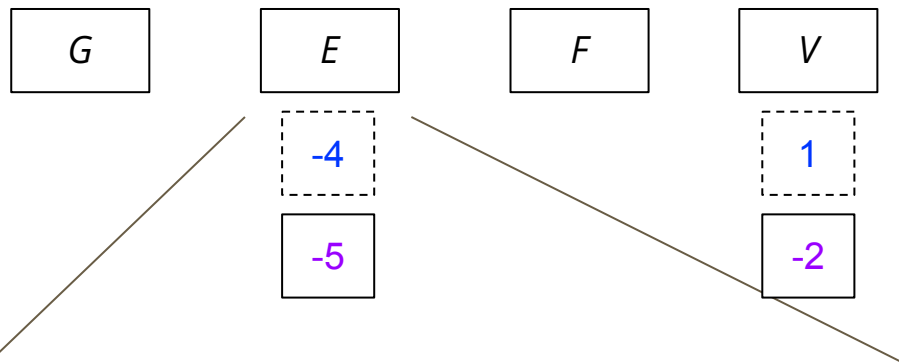
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            { V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                { V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                { V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                { E(i,j)
                { F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$





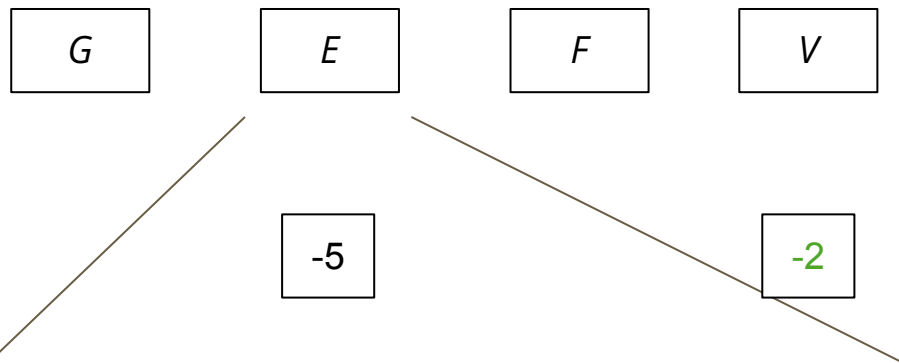
E	A	T	C	G	G	C	
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	
A	$-3_V$	$-6_V$	$-7_V$	$-8_V$	$-9_V$	$-10_V$	$-11_V$
G	$-4_E$	$-1_V$	$-4_V$	$-5_V$	$-6_V$	$-7_V$	$-8_V$
C	$-5_E$	$-2_E$					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



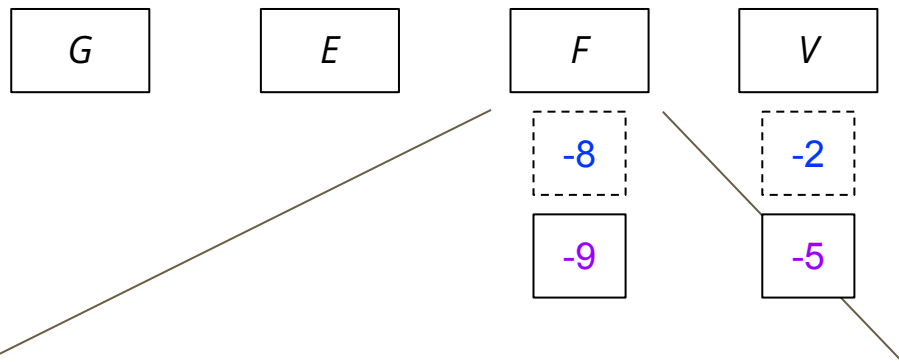
E	A	T	C	G	G	C	
$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	
A	$-3_V$	$-6_V$	$-7_V$	$-8_V$	$-9_V$	$-10_V$	$-11_V$
G	$-4_E$	$-1_V$	$-4_V$	$-5_V$	$-6_V$	$-7_V$	$-8_V$
C	$-5_E$	$-2_E$	$-2_V$				

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



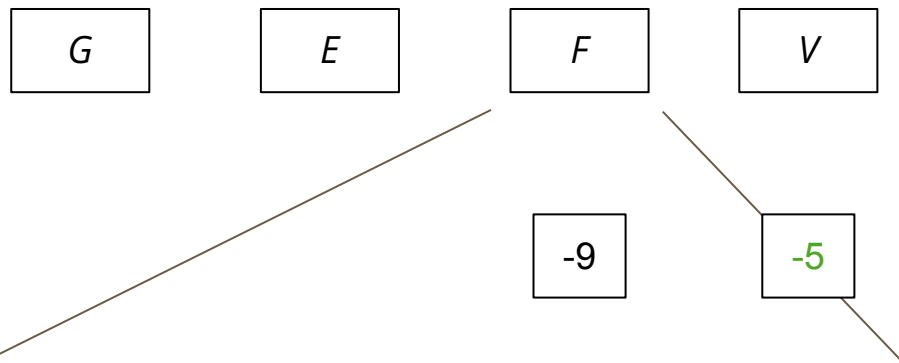
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$
C	$-\infty$	$-8_V$				

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$



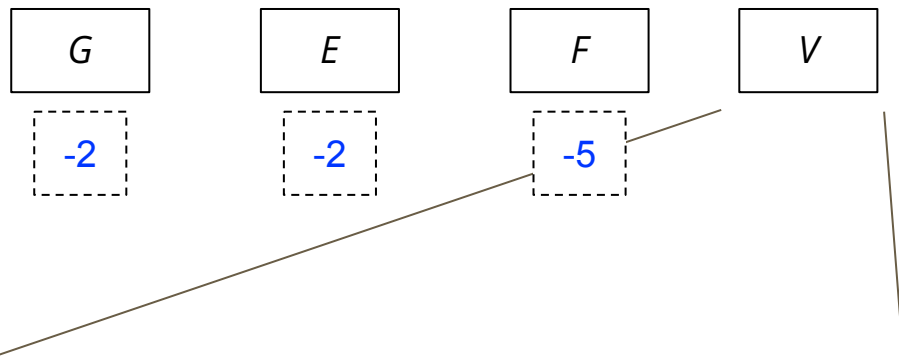
F	A	T	C	G	G	C
$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$
C	$-\infty$	$-8_V$	$-5_V$			

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



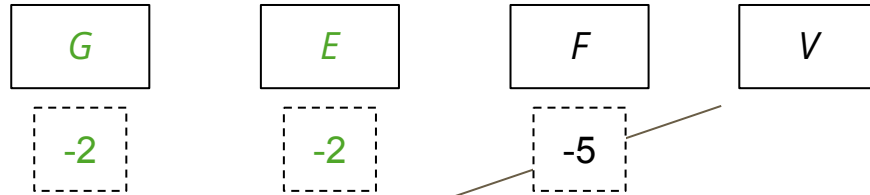
V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>					

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if xi = yj
            V(i-1,j-1) - μ if xi ≠ yj
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2 \quad \gamma = 2$   
 $\mu = 1 \quad \tau = 1$



V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$	$-1_E$	$1_G$	$-2_{G/F}$	$0_G$	$-1_G$	$-4_F$
C	$-5_E$	$-2_E$	$-2_{G/E}$				

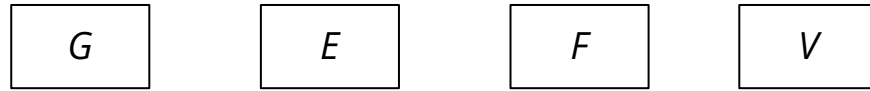
```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$     $\gamma = 2$   
 $\mu = 1$     $\tau = 1$





<b>G</b>		<b>A</b>	<b>T</b>	<b>C</b>	<b>G</b>	<b>G</b>	<b>C</b>
	-	-	-	-	-	-	-
<b>A</b>	-	2	-4	-5	-6	-7	-8
<b>G</b>	-	-4	1	-2	0	-1	-5
<b>C</b>	-	-5	-2	3	-3	-1	1

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$





E		A	T	C	G	G	C
	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
A	$-3_V$	$-6_V$	$-7_V$	$-8_V$	$-9_V$	$-10_V$	$-11_V$
G	$-4_E$	$-1_V$	$-4_V$	$-5_V$	$-6_V$	$-7_V$	$-8_V$
C	$-5_E$	$-2_E$	$-2_V$	$-5_V$	$-3_V$	$-4_V$	$-7_V$

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



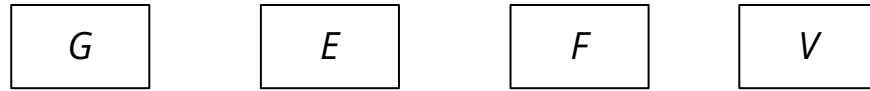
F		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$



V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$	$-1_E$	$1_G$	$-2_{G/F}$	$0_G$	$-1_G$	$-4_F$
C	$-5_E$	$-2_E$	$-2_{G/E}$	$3_G$	$0_F$	$-1_{G/F}$	$1_G$

```

V(0,0) ← 0
for j ← 1 to n do
  | V(0,j) = E(0,j) ← -γ - jτ
end
for i ← 1 to m do
  | V(i,0) = F(i,0) ← -γ - iτ
end
for i ← 1 to m, j ← 1 to n do
  G(i,j) ← { V(i-1,j-1) + α if x_i = y_j
            V(i-1,j-1) - μ if x_i ≠ y_j
  E(i,j) ← max { E(i,j-1) - τ
                V(i,j-1) - γ - τ
  F(i,j) ← max { F(i-1,j) - τ
                V(i-1,j) - γ - τ
  V(i,j) ← max { G(i,j)
                E(i,j)
                F(i,j)
end

```

$\alpha = 2$	$\gamma = 2$
$\mu = 1$	$\tau = 1$

# Traceback

We use the backpointers in our matrices to reconstruct our alignment.

At each position, we can recover the single-letter alignment of the prior two characters based on which matrix produced our maximum score.

Starting from  $V(m,n)$ , at every  $V(i,j)$ :

- If  $\text{argmax} = G \rightarrow$  recover a match/mismatch; recurse on  $V(i-1, j-1)$
- If  $\text{argmax} = E \rightarrow$  recover a gap in  $X$ ; follow the backpointers of  $E(i,j)$  recursively, inserting gaps in  $X$  until we return to  $V$ ; recurse
- If  $\text{argmax} = F \rightarrow$  recover a gap in  $Y$ ; follow the backpointers of  $F(i,j)$  recursively, inserting gaps in  $Y$  until we return to  $V$ ; recurse

Y  
sequence

X sequence

V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>

Score:

**+1**

V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>



Score:

**+1**

<b>V</b>		A	T	C	G	G	<b>C</b>
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
<b>C</b>	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>



C  
C

Score:  
+1

V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>



C  
C

Score:  
+1



<b>V</b>		A	T	C	G	<b>G</b>	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
<b>G</b>	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>

**G**  
**G**

C  
C

Score:  
**+1**

Here you hit the backpointer referencing  $F$  matrix. So you must continue backtracking through the  $F$  matrix.

V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$	$-1_E$	$1_G$	$-2_{G/F}$	$0_G$	$-1_G$	$-4_F$
C	$-5_E$	$-2_E$	$-2_{G/E}$	$3_G$	$0_F$	$-1_{G/F}$	$1_G$



G C  
G C

Score:  
+1

We switched to the  $F$  matrix.

<b>F</b>		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$



G C  
G C

Score:  
**+1**

<b>F</b>		A	T	C	<b>G</b>	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	<b><math>-3_F</math></b>	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$

**G**  
 -

G      C  
 G      C

Score:  
+1

<b>F</b>		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$



G G C  
 - G C

Score:

+1

<b>F</b>		A	T	<b>C</b>	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$

**C**  
 -

G

-

G

G

C

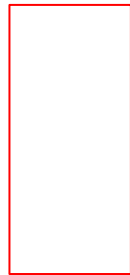
C

Score:

+1

Here you hit the backpointer referencing  $V$  matrix. So you must continue backtracking through the  $V$  matrix

F		A	T	C	G	G	C
	$-\infty$	$-3_V$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-\infty$	$-6_V$	$-1_V$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-\infty$	$-7_V$	$-4_V$	$-2_V$	$-3_F$	$-3_V$	$-4_V$
C	$-\infty$	$-8_V$	$-5_V$	$-5_V$	$0_V$	$-1_F$	$-2_F$



C      G      G      C  
 -      -      G      C

Score:  
 +1

We switched to the  $V$  matrix.

V		A	T	C	G	G	C
	0	$-3_F$	$-4_F$	$-5_F$	$-6_F$	$-7_F$	$-8_F$
A	$-3_E$	$2_G$	$-1_F$	$-2_F$	$-3_F$	$-4_F$	$-5_F$
G	$-4_E$	$-1_E$	$1_G$	$-2_{G/F}$	$0_G$	$-1_G$	$-4_F$
C	$-5_E$	$-2_E$	$-2_{G/E}$	$3_G$	$0_F$	$-1_{G/F}$	$1_G$

T
-

C	G	G	C
-	-	G	C

Score:  
+1



V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>



T C G G C  
 - - - G C

Score:  
 +1

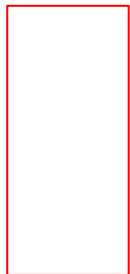
V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>

A  
A

T C G G C  
- - - G C

Score:  
+1

V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>



A T C G G C  
 A - - - G C

Score:  
 +1

V		A	T	C	G	G	C
	0	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>	-6 <sub>F</sub>	-7 <sub>F</sub>	-8 <sub>F</sub>
A	-3 <sub>E</sub>	2 <sub>G</sub>	-1 <sub>F</sub>	-2 <sub>F</sub>	-3 <sub>F</sub>	-4 <sub>F</sub>	-5 <sub>F</sub>
G	-4 <sub>E</sub>	-1 <sub>E</sub>	1 <sub>G</sub>	-2 <sub>G/F</sub>	0 <sub>G</sub>	-1 <sub>G</sub>	-4 <sub>F</sub>
C	-5 <sub>E</sub>	-2 <sub>E</sub>	-2 <sub>G/E</sub>	3 <sub>G</sub>	0 <sub>F</sub>	-1 <sub>G/F</sub>	1 <sub>G</sub>

A     T     C     G     G     C  
A     -     -     -     G     C

Score:  
+1

# Results:

A	T	C	G	G	C
A	-	-	-	G	C

...is our optimal alignment with score **+1!**