Knuth-Morris-Pratt Algorithm

CS181 Fall 2020

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Overview of Knuth-Morris-Pratt (KMP)

- The Knuth-Morris-Pratt (KMP) algorithm is a pattern-matching algorithm; it finds all occurrences of a pattern $P$ of length $p$ in a text $T$ of length $t$
- It takes advantage of the failure function $f$ on the pattern $P$ to search in linear time $O(p + t)$!
  - The general idea is that after we’ve seen a character in $T$ once, we should already be able to tell whether the pattern could start there, even if we never explicitly attempted to match $P_1$ directly to $T_j$
- We’ve already seen the algorithm and pseudocode for constructing the failure function, so we’ll focus on KMP here using a similar example
Definitions

● Inputs:
  ○ Text $T$, indexed by $j$ from 1 to $t$
  ○ Pattern $P$, indexed by $i$ from 1 to $p$

● Output:
  ○ A list of positions $k$, where $T_{k:k+p} = P$

● Failure function, $f$
  ○ A table of $p$ entries, where each entry $f(i)$ is the length of the longest proper suffix of $P_{1:i}$ which is also a proper prefix of $P$
  ○ See previous slide deck for a more detailed explanation
The Algorithm

1. Calculate the failure function \( f \) for the pattern \( P \)
2. Construct a skeleton DFA which accepts \( P \) and includes transitions based on \( f \)
3. Initialize the skeleton DFA to state 0 and the \( T \) pointer to 1
4. Iterate through the text \( T \)

**Here we show a version of the pseudocode which conceptualizes KMP with an accepting skeleton DFA. In practice, the skeleton DFA behavior can also be achieved using only the pattern \( P \), the failure function \( f \), and a pointer \( i \) which indexes symbols in \( P \) rather than states in \( M \).**

calculate \( f(i) \) for \( 1 \leq i \leq p \)
construct a skeleton DFA \( M \) for \( P \) using \( f \)
\( M \) starts in state \( M_0 \)
i := current state in \( M \) (updated with transitions)
j \( \leftarrow 1 \)
while \( j \leq t \) do
    if \( T_j = P_{i+1} \) then
        \( j \leftarrow j + 1 \)
        \( M \) enters state \( M_{i+1} \)
        if \( M \) is in state \( M_p \) then
            record \( (j - p) \)
            \( M \) enters state \( M_{f(p)} \)
        end
    else
        \( M \) enters state \( M_{f(i)} \)
        if \( M \) is in state \( M_0 \) and \( T_j \neq P_{i+1} \) then
            \( j \leftarrow j + 1 \)
        end
    end
end
An Example

\( T = \text{aabbabaabaabaca} \)

\( P = \text{abaabc} \)

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    if \( M \) is in state \( M_0 \) and \( T_j \neq P_{i+1} \) then
      \( j \leftarrow j + 1 \)
    end
  end
end
**See previous set of slides for exactly how we constructed this!**
calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA $M$ for $P$ using $f$

$M$ starts in state $M_0$

$i :=$ current state in $M$ (updated with transitions)

$j \leftarrow 1$

while $j \leq t$ do

if $T_j = P_{i+1}$ then

$j \leftarrow j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

record $(j - p)$

$M$ enters state $M_{f(p)}$

end

else

$M$ enters state $M_{f(i)}$

if $M$ is in state $M_0$ and $T_j \neq P_{i+1}$ then

$j \leftarrow j + 1$

end

end
calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA $M$ for $P$ using $f$

$M$ starts in state $M_0$
i := current state in $M$ (updated with transitions)
j $\leftarrow$ 1

while $j$ $\leq t$ do
  if $T_j = P_{i+1}$ then
    $j$ $\leftarrow$ $j + 1$
    $M$ enters state $M_{i+1}$
    if $M$ is in state $M_p$ then
      record $(j - p)$
      $M$ enters state $M_{f(p)}$
    end
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    if $M$ is in state $M_0$ and $T_j \neq P_{i+1}$ then
      $j$ $\leftarrow$ $j + 1$
    end
  end
end

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>$f(i)$</td>
<td>0</td>
<td>0</td>
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<td>2</td>
<td>0</td>
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      $j \leftarrow j + 1$
    end
  end
end
construct $f(i)$ for $1 \leq i \leq p$.

$M$ starts in state $M_0$.

$i \leftarrow i + 1$

else

if $M$ is in state $M_t$ and $T_j \neq P_{i+1}$ then

if $M$ is in state $M_t$ then

$M$ enters state $M_{t+1}$

end

$M$ enters state $M_{t+1}$

end

end

end

$i = 0, 1, 2, 3, 4, 5, 6$
calculate $f(i)$ for $1 \leq i \leq p$

construct a skeleton DFA $M$ for $P$ using $f$

$M$ starts in state $M_0$

$i :=$ current state in $M$ (updated with transitions)

$j \leftarrow 1$

while $j \leq t$ do

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if $M$ is in state $M_p$ then

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while $j \leq t$ do
  if $T_j = P_{i+1}$ then
    $j \leftarrow j + 1$
    $M$ enters state $M_{i+1}$
    \textbf{if} $M$ is in state $M_p$ \textbf{then}
      record $(j - p)$
      $M$ enters state $M_{f(p)}$
    \textbf{end}
  \textbf{else}
    $M$ enters state $M_{f(i)}$
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\( M \) starts in state \( M_0 \) \\
i := current state in \( M \) (updated with transitions) \\
j := 0 \\
while \( j \leq t \) do \\
  if \( T_j = P_{i+1} \) then \\
    j := j + 1 \\
    M enters state \( M_{i+1} \) \\
    if \( M \) is in state \( M_p \) then \\
      record \( (j - p) \) \\
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$j \leftarrow 1$

while $j \leq t$

if $T_j = P_{i+1}$ then

\begin{align*}
    j &\leftarrow j + 1 \\
    M &\text{ enters state } M_{i+1} \hspace{1cm} \text{if } M \text{ is in state } M_p \text{ then} \\
    &\text{ record } (j - p) \\
    &\text{ } M \text{ enters state } M_{f(p)} \\
    \end{align*}

else

\begin{align*}
    M &\text{ enters state } M_{f(i)} \\
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j ← 1
while $j \leq t$ do
    if $T_j = P_{i+1}$ then
        j ← j + 1
        $M$ enters state $M_{i+1}$
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| $f(i)$ | 0 | 0 | 1 | 1 | 2 | 0 |
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calculate \( f(i) \) for \( 1 \leq i \leq p \)

construct a skeleton DFA \( M \) for \( P \) using \( f \)

\( M \) starts in state \( M_0 \)

\( i := \) current state in \( M \) (updated with transitions)

\( j \leftarrow 1 \)

while \( j \leq t \) do

if \( T_j = P_{i+1} \) then

\( j \leftarrow j + 1 \)

\( M \) enters state \( M_{i+1} \)

if \( M \) is in state \( M_p \) then

record \((j - p)\)

\( M \) enters state \( M_{f(p)} \)

end

else

\( M \) enters state \( M_{f(i)} \)

if \( M \) is in state \( M_0 \) and \( T_j \neq P_{i+1} \) then

\( j \leftarrow j + 1 \)

end

end

end

---

\( f(i) \)

\begin{array}{cccccc}
0 & 0 & 1 & 1 & 2 & 0 \\
\end{array}
calculate $f(i)$ for $1 \leq i \leq p$
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      $M$ enters state $M_{f(p)}$
    end
  else
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$j \leftarrow 1$

while $j \leq t$ do

if $T_j = P_{i+1}$ then

$j \leftarrow j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

record $(j - p)$

$M$ enters state $M_{f(p)}$

end

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while $j \leq t$ do

if $T_j = P_{i+1}$ then

$j \leftarrow j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

record $(j - p)$

$M$ enters state $M_{f(p)}$

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record \( (j - p) \)

\( M \) enters state \( M_{f(p)} \)

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$M$ starts in state $M_0$
i := current state in $M$ (updated with transitions)
j ← 1

while $j \leq t$ do

if $T_j = P_{i+1}$ then

j ← $j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

记录 $(j-p)$

$M$ enters state $M_{f(p)}$

end

else

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$j \leftarrow j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

record $(j - p)$

$M$ enters state $M_{f(p)}$

end

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      record \( (j - p) \)
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      \( j \leftarrow j + 1 \)
    end
  end
end

\[
\begin{array}{ccccccc}
  f(i) & 0 & 0 & 1 & 1 & 2 & 0 \\
\end{array}
\]
calculate $f(i)$ for $1 \leq i \leq p$

construct a skeleton DFA $M$ for $P$ using $f$

$M$ starts in state $M_0$

$i :=$ current state in $M$ (updated with transitions)

$j \leftarrow 1$

while $j \leq t$ do

if $T_j = P_{i+1}$ then

\[ j \leftarrow j + 1 \]

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

\[ j \leftarrow j - p \]

$M$ enters state $M_{f(p)}$

end

else

$M$ enters state $M_{f(i)}$

if $M$ is in state $M_0$ and $T_j \neq P_{i+1}$ then

\[ j \leftarrow j + 1 \]

end

end

end

\[
f(i) \quad 0 \quad 0 \quad 1 \quad 1 \quad 2 \quad 0
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      record $(j - p)$
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if $T_j = P_{i+1}$ then

\[ j \leftarrow j + 1 \]

$M$ enters state $M_{i+1}$

\[ \textbf{if } M \text{ is in state } M_p \text{ then} \]

\[ \text{record } (j - p) \]

$M$ enters state $M_{f(p)}$

end

else

$M$ enters state $M_{f(i)}$

\[ \text{if } M \text{ is in state } M_0 \text{ and } T_j \neq P_{i+1} \text{ then} \]

\[ j \leftarrow j + 1 \]

end

end

\[ f(i) \]

\[ \begin{array}{ccccccc}
0 & 0 & 1 & 1 & 2 & 0 & 8
\end{array} \]
calculate $f(i)$ for $1 \leq i \leq p$

construct a skeleton DFA $M$ for $P$ using $f$

$M$ starts in state $M_0$

$i := \text{current state in } M$ (updated with transitions)

$j \leftarrow 1$

while $j \leq t$ do

if $T_j = P_{i+1}$ then

$j \leftarrow j + 1$

$M$ enters state $M_{i+1}$

if $M$ is in state $M_p$ then

record $(j - p)$

$M$ enters state $M_{f(p)}$

end

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while $j \leq t$ do
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        $j \leftarrow j + 1$
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            record $(j - p)$
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construct a skeleton DFA $M$ for $P$ using $f$
$M$ starts in state $M_0$
i := current state in $M$ (updated with transitions)
j := 1
while $j \leq t$ do
  if $T_j = P_{i+1}$ then
    \begin{align*}
      j &\leftarrow j + 1 \\
      M &\text{ enters state } M_{i+1}
    \end{align*}
    if $M$ is in state $M_p$ then
      record $(j - p)$
      $M$ enters state $M_{f(p)}$
    end
  else
    $M$ enters state $M_{f(i)}$
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        j &\leftarrow j + 1
      \end{align*}
    end
  end
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if $T_j = P_{i+1}$ then
  $j := j + 1$
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  if $M$ is in state $M_p$ then
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      $j \leftarrow j + 1$
    end
  end
end
Results:

The pattern $P = \text{“abaabc”}$ occurs once in $T = \text{“aabbabaabaabca”}$ starting at position 8.