# Viterbi Algorithm

CS 181 Fall 2021

# Definition: Inputs and Outputs

- Inputs <Θ, λ = (n, m, A, B, π)>
  - Θ = a sequence of observations
  - λ = a Hidden Markov Model
    - n = the number of hidden states, S = {$s_1$, $s_2$, ... $s_n$}
    - m = the number of observation symbols, V = {$v_1$, $v_2$, ... $v_m$}
    - A = the transition probability distribution $A = \{a_{ij} = \mathbb{P}(q_{t+1} = s_j | q_t = s_i)\}$
      - The (*i*, *j*)-th entry is the probability that the HMM is in state *j* at time *t*+1, given that it was in state *i* at time *t*
    - B = the emission probability distribution $B = \{b_j(k) = \mathbb{P}(v_k \text{ at time } t | q_t = s_j)\}$
      - The (*j*, *k*)-th entry is the probability that the HMM emits symbol *k* at time t, given that is was in state *j* at time *t*)
    - π = the initial state distribution $\pi = \{\pi_i = \mathbb{P}(q_1 = s_i)\}$
- Outputs: *Q* for which $\mathbb{P}(Q|\Theta)$ is maximal
  - The most likely sequence of states (and the probability of observing it)

# Definition: Auxiliary Data Structures

- $\delta$ = Scoring matrix $\delta = \{\delta_t(i) = \underset{q_1 \cdots q_{t-1}}{\text{MAX}} \{\mathbb{P}(\theta_1 \ldots \theta_t, q_t = s_i)\}$
    - The (*i*, *t*)-th entry gives the maximum probability of observing $\theta_1, \ldots, \theta_t$ along any sequence of states and ending in state *i* at time *t*
- $\psi$ = Backtracking matrix
    - The (*i*, *t*)-th entry gives the state at time *t*-1 which produces the maximum probability path ending in state *i* at time *t*

# The Algorithm

1) Initialize the matrices
2) Apply the recurrence relations to fill each matrix
3) Compute the maximum probability
4) Initialize the backtracking process
5) Complete the backtracking
6) Output $p^*$ and $q_1^*, \ldots, q_T^*$

/* Initialization

**for** $i \leftarrow 1$ **to** $n$ **do**

$\quad \delta_1(i) \leftarrow \pi_i b_i(\theta_1)$

$\quad \psi_1(i) \leftarrow 0$

**end**

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

$\quad$ **for** $j \leftarrow 1$ **to** $n$ **do**

$\quad\quad \delta_t(j) \leftarrow \underset{1 \le i \le n}{\mathrm{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$

$\quad\quad \psi_t(j) \leftarrow \underset{1 \le i \le n}{\mathrm{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$

$\quad$ **end**

**end**

/* Termination

$p^* \leftarrow \underset{1 \le i \le n}{\mathrm{MAX}}[\delta_T(i)]$

$q_T^* \leftarrow \underset{1 \le i \le n}{\mathrm{ARGMAX}}[\delta_T(i)]$

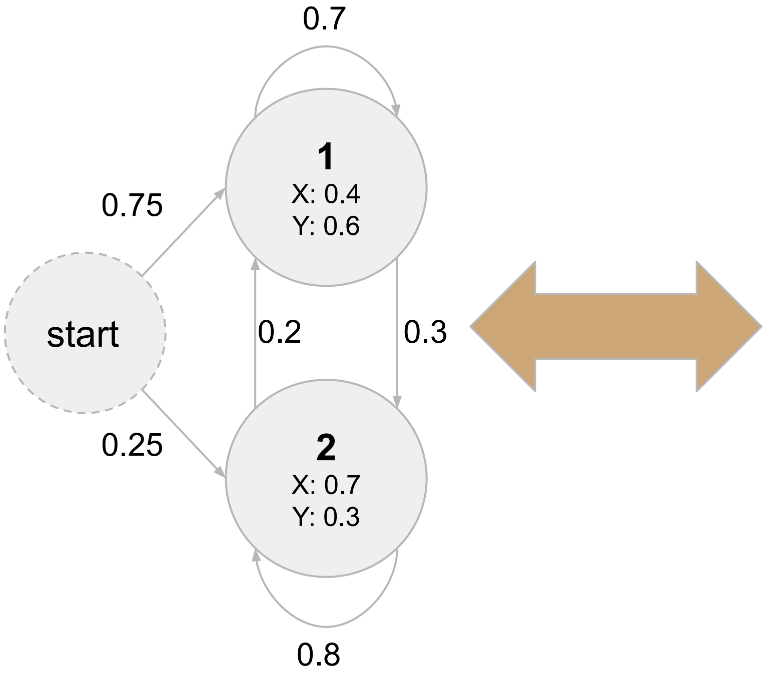
/* Backtracking

**for** $t \leftarrow T-1$ **to** $1$ **do**

$\quad q_t^* \leftarrow \psi_{t+1}(q_{t+1}^*)$

**end**

# An Example




Observation sequence, θ: **XYYXX**

| **π** | 1 | 2 |
|---|---|---|
| $P(q_1)$ | 0.75 | 0.25 |

| **A** | 1 | 2 |
|---|---|---|
| 1 | 0.7 | 0.3 |
| 2 | 0.2 | 0.8 |

| **B** | X | Y |
|---|---|---|
| 1 | 0.4 | 0.6 |
| 2 | 0.7 | 0.3 |

# XYYXX

```
/* Initialization
```
**for** $i \leftarrow 1$ **to** $n$ **do**

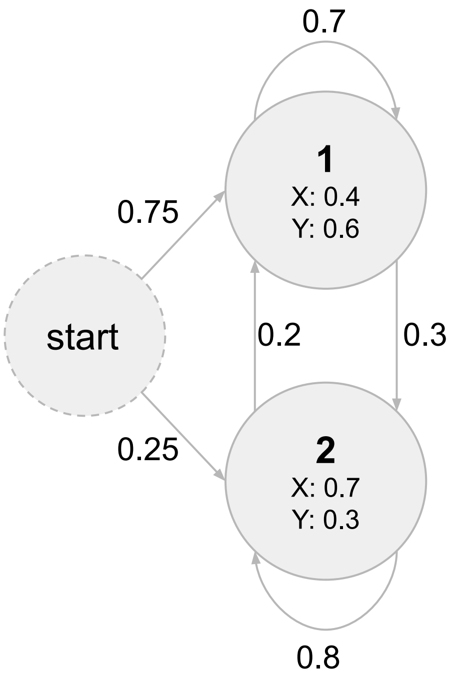
$\quad \delta_1(i) \leftarrow \pi_i b_i(\theta_1)$

$\quad \psi_1(i) \leftarrow 0$

**end**

| δ | 1 | 2 |
|---|---|---|
| t = 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

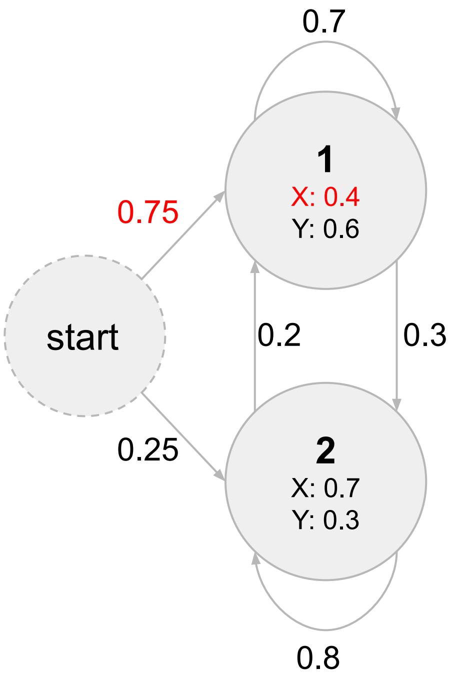


XYYXX

/* Initialization
for $i \leftarrow 1$ to $n$ do
  $\delta_1(i) \leftarrow \pi_i b_i(\theta_1)$
  $\psi_1(i) \leftarrow 0$
end

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

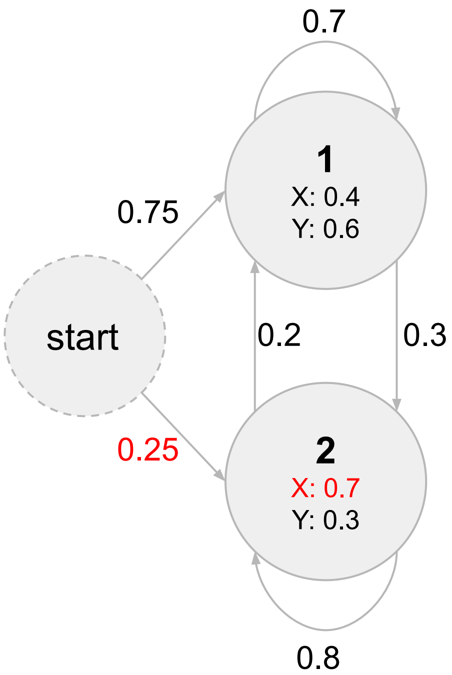


# XYYXX

```
/* Initialization
for i ← 1 to n do
    δ_1(i) ← π_i b_i(θ_1)
    ψ_1(i) ← 0
end
```

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

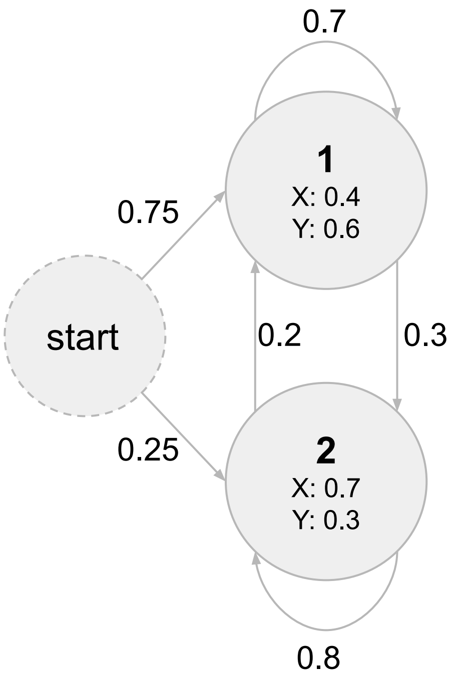


# XYYXX

```
/* Initialization
for i ← 1 to n do
    δ_1(i) ← π_i b_i(θ_1)
    ψ_1(i) ← 0
end
```

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



# XYYXX

j=1: max(0.21, 0.035)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

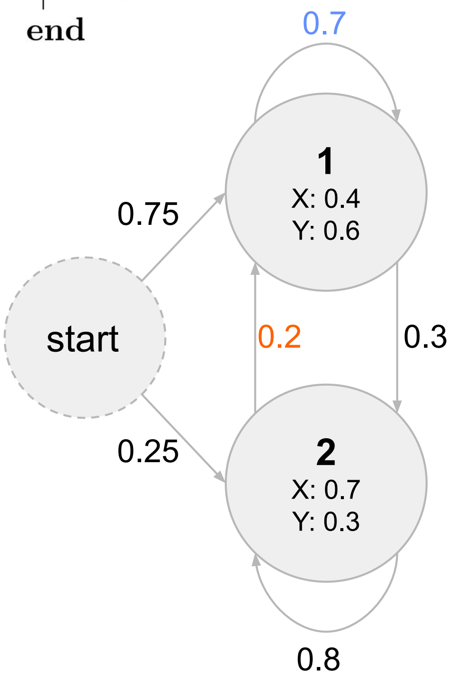
  **for** $j \leftarrow 1$ **to** $n$ **do**

    $\delta_t(j) \leftarrow \underset{1 \le i \le n}{\mathrm{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$

    $\psi_t(j) \leftarrow \underset{1 \le i \le n}{\mathrm{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$

  **end**

**end**



# XYYXX

j=1: max(0.21, 0.035)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | |
| 3 | | |
| 4 | | |
| 5 | | |

```
/* Recurrence
```

**for** $t \leftarrow 2$ **to** $T$ **do**

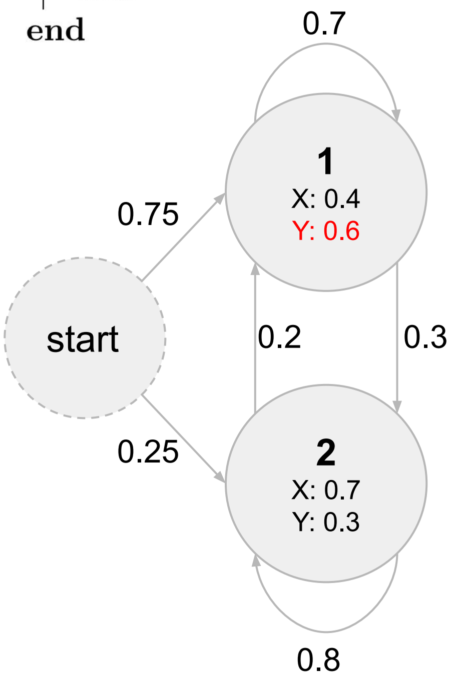
  **for** $j \leftarrow 1$ **to** $n$ **do**

$$\delta_t(j) \leftarrow \underset{1\leq i\leq n}{\text{MAX}}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \underset{1\leq i\leq n}{\text{ARGMAX}}[\delta_{t-1}(i)\cdot a_{ij}]$$

  **end**

**end**



# XYYXX

j=1: max(0.09, 0.14)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | |
| 3 | | |
| 4 | | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

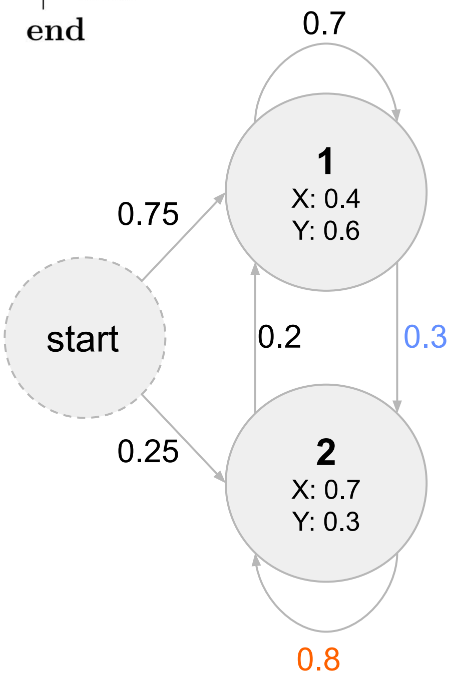
  **for** $j \leftarrow 1$ **to** $n$ **do**

    $\delta_t(j) \leftarrow \max_{1\le i\le n}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$

    $\psi_t(j) \leftarrow \mathrm{ARGMAX}_{1\le i\le n}[\delta_{t-1}(i) \cdot a_{ij}]$

  **end**

**end**



# XYYXX

j=1: max(0.09, 0.14)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | | |
| 4 | | |
| 5 | | |

```
/* Recurrence
```

**for** $t \leftarrow 2$ **to** $T$ **do**

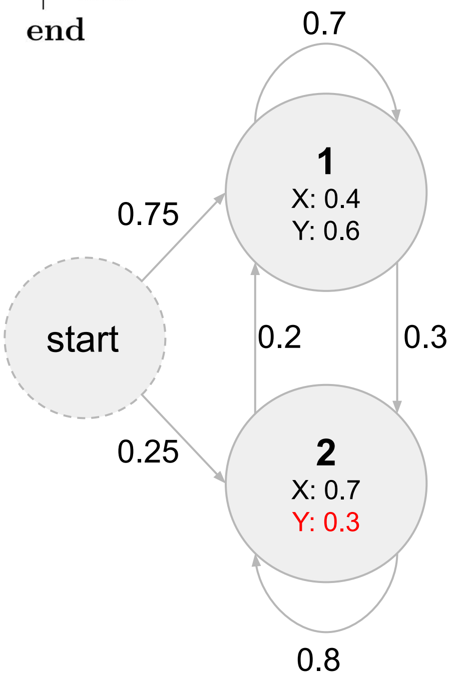
  **for** $j \leftarrow 1$ **to** $n$ **do**

$$\delta_t(j) \leftarrow \underset{1 \le i \le n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \underset{1 \le i \le n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$$

  **end**

**end**



# XYYXX

j=1: max(0.0882, 0.0084)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | | |
| 4 | | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

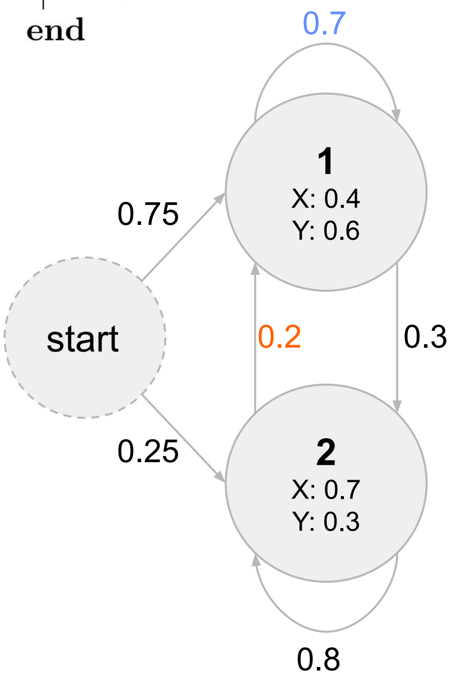
  **for** $j \leftarrow 1$ **to** $n$ **do**

$$\delta_t(j) \leftarrow \underset{1\leq i\leq n}{\mathrm{MAX}}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \underset{1\leq i\leq n}{\mathrm{ARGMAX}}[\delta_{t-1}(i)\cdot a_{ij}]$$

  **end**

**end**



# XYYXX

j=1: max(0.0882, 0.0084)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | |
| 4 | | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

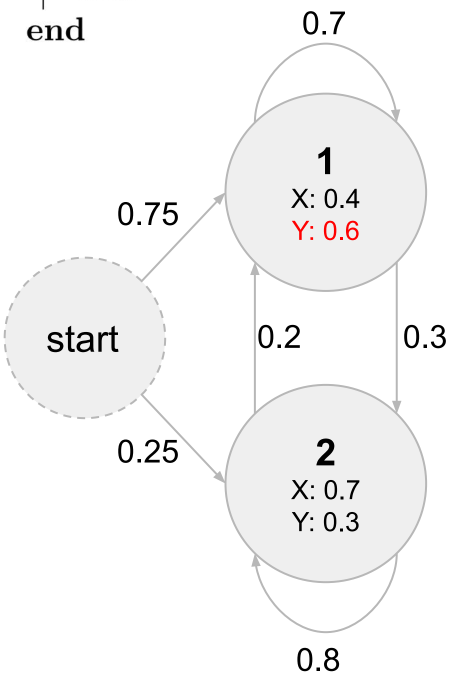
  **for** $j \leftarrow 1$ **to** $n$ **do**

$$\delta_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$$

  **end**

**end**



# XYYXX

j=1: max(0.0378, 0.0336)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | |
| 4 | | |
| 5 | | |

```
/* Recurrence
```

**for** $t \leftarrow 2$ **to** $T$ **do**

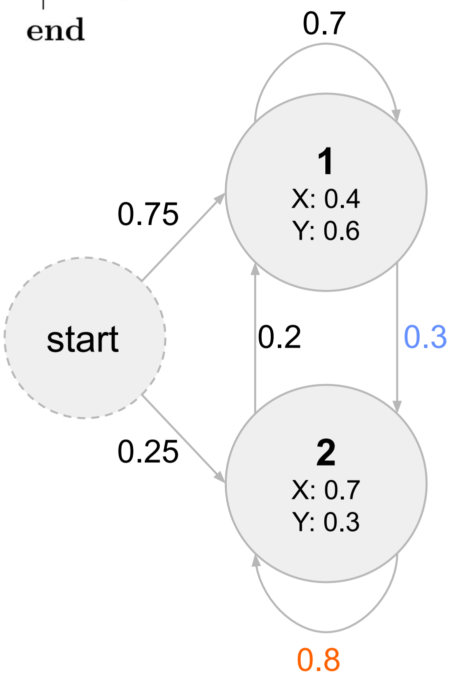
  **for** $j \leftarrow 1$ **to** $n$ **do**

    $\delta_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$

    $\psi_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$

  **end**

**end**



# XYYXX

j=1: max(0.0378, 0.0336)

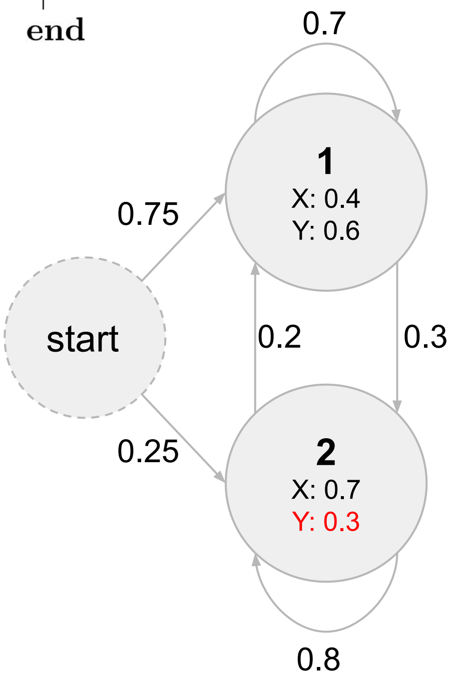
| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | | |
| 5 | | |

```
/* Recurrence
for t ← 2 to T do
    for j ← 1 to n do
        δ_t(j) ← MAX_{1≤i≤n}[δ_{t-1}(i) · a_ij] · b_j(θ_t)
        ψ_t(j) ← ARGMAX_{1≤i≤n}[δ_{t-1}(i) · a_ij]
    end
end
```

$$\delta_t(j) \leftarrow \max_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \operatorname{ARGMAX}_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]$$



# XYYXX

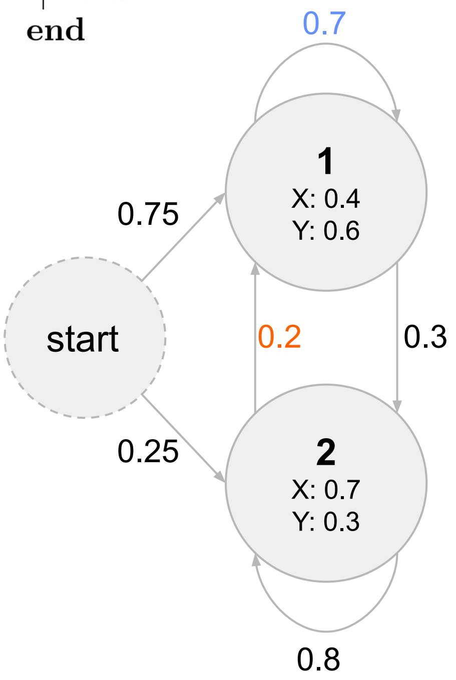
j=1: max(0.0370, 0.0027)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**
  **for** $j \leftarrow 1$ **to** $n$ **do**
    $\delta_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$
    $\psi_t(j) \leftarrow \underset{1 \leq i \leq n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$
  **end**
**end**



# XYYXX

j=1: max(0.0370, 0.0027)

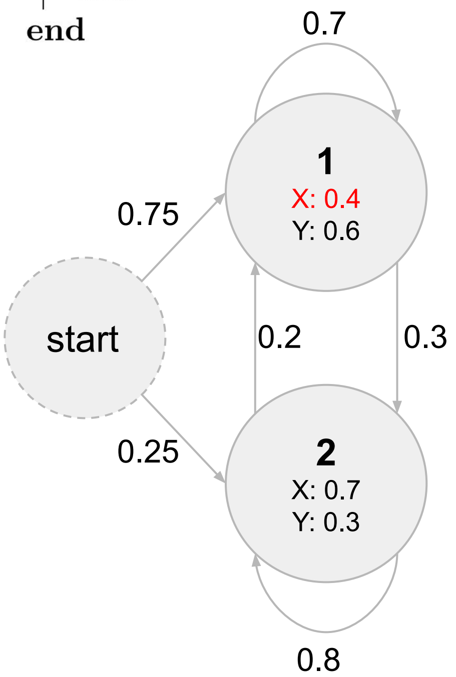
| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | |
| 5 | | |

```
/* Recurrence
for t ← 2 to T do
    for j ← 1 to n do
        δ_t(j) ← MAX_{1≤i≤n}[δ_{t−1}(i) · a_ij] · b_j(θ_t)
        ψ_t(j) ← ARGMAX_{1≤i≤n}[δ_{t−1}(i) · a_ij]
    end
end
```

$$\delta_t(j) \leftarrow \max_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \operatorname{ARGMAX}_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]$$



# XYYXX

j=1: max(0.0159, 0.0091)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | |
| 5 | | |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**

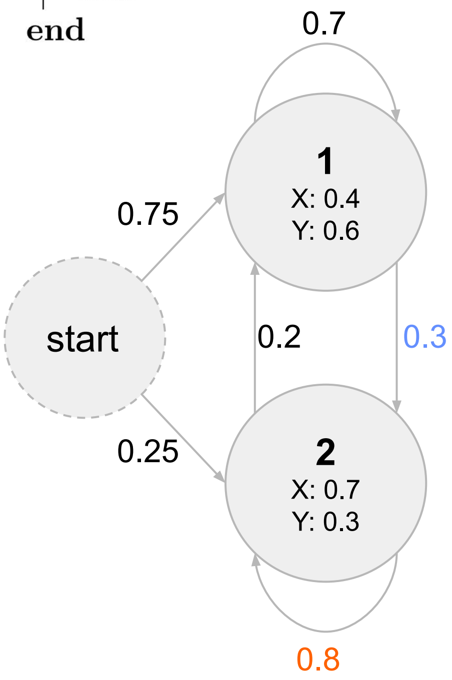
  **for** $j \leftarrow 1$ **to** $n$ **do**

    $\delta_t(j) \leftarrow \underset{1 \le i \le n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$

    $\psi_t(j) \leftarrow \underset{1 \le i \le n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$

  **end**

**end**



XYYXX

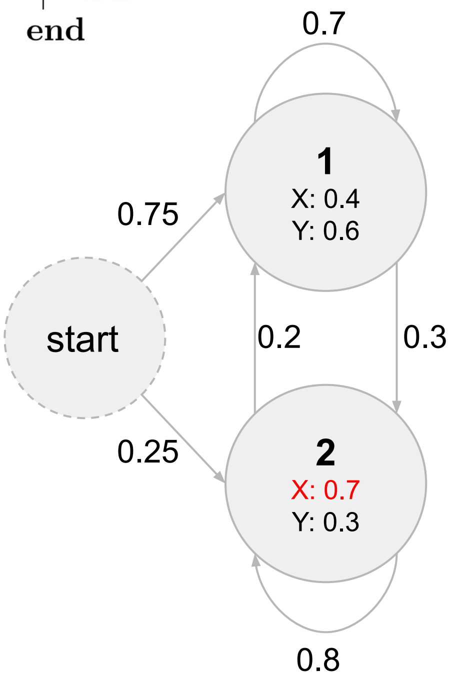
j=1: max(0.0159, 0.0091)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | | |

```
/* Recurrence
```

**for** $t \leftarrow 2$ **to** $T$ **do**
  **for** $j \leftarrow 1$ **to** $n$ **do**
    $\delta_t(j) \leftarrow \underset{1 \le i \le n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$
    $\psi_t(j) \leftarrow \underset{1 \le i \le n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$
  **end**
**end**



# XYYXX

j=1: max(0.0104, 0.0022)

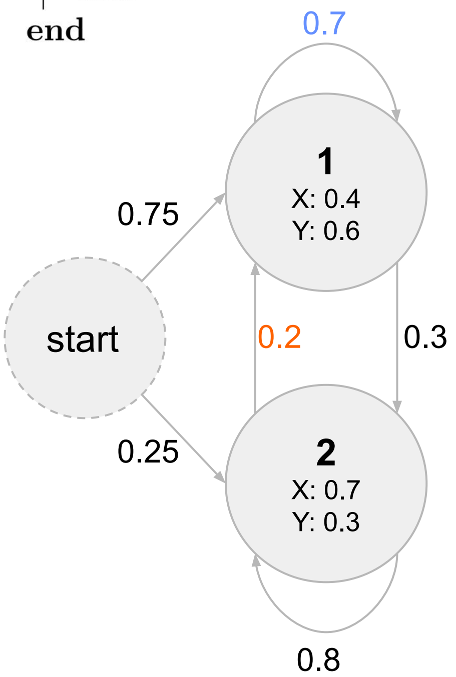
| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | | |

```
/* Recurrence
for t ← 2 to T do
    for j ← 1 to n do
        δ_t(j) ← MAX_{1≤i≤n}[δ_{t−1}(i) · a_ij] · b_j(θ_t)
        ψ_t(j) ← ARGMAX_{1≤i≤n}[δ_{t−1}(i) · a_ij]
    end
end
```

$$\delta_t(j) \leftarrow \max_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \operatorname{ARGMAX}_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]$$



# XYYXX

j=1: max(0.0104, 0.0022)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | |

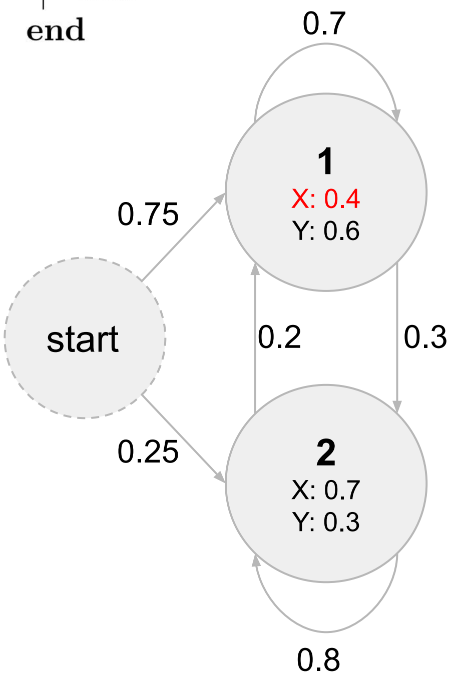
```
/* Recurrence
```

**for** $t \leftarrow 2$ **to** $T$ **do**
  **for** $j \leftarrow 1$ **to** $n$ **do**

$$\delta_t(j) \leftarrow \underset{1 \le i \le n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \underset{1 \le i \le n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$$

  **end**
**end**



# XYYXX

j=1: max(0.0044, 0.0089)

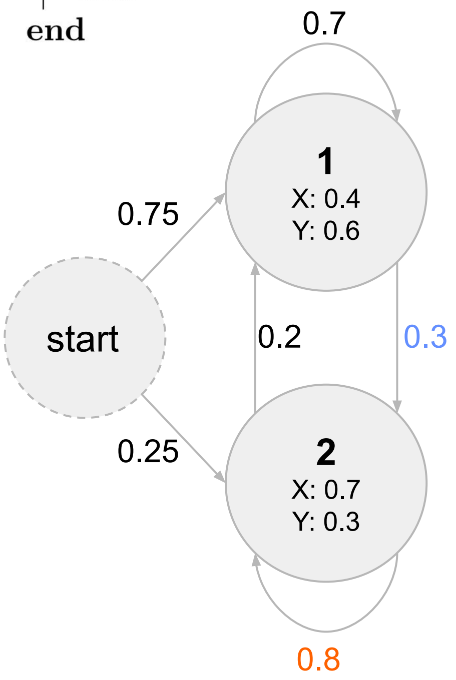
| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | |

```
/* Recurrence
for t ← 2 to T do
    for j ← 1 to n do
        δ_t(j) ← MAX_{1≤i≤n}[δ_{t-1}(i) · a_ij] · b_j(θ_t)
        ψ_t(j) ← ARGMAX_{1≤i≤n}[δ_{t-1}(i) · a_ij]
    end
end
```

$$\delta_t(j) \leftarrow \max_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]\cdot b_j(\theta_t)$$

$$\psi_t(j) \leftarrow \operatorname{ARGMAX}_{1\le i\le n}[\delta_{t-1}(i)\cdot a_{ij}]$$



# XYYXX

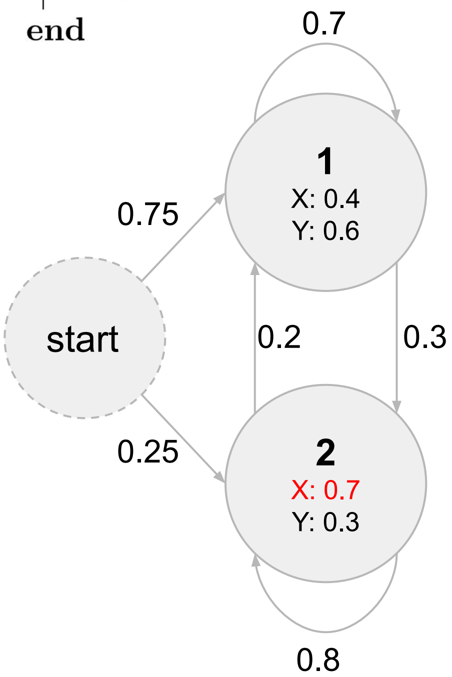
j=1: max(0.0044, 0.0089)

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

/* Recurrence

**for** $t \leftarrow 2$ **to** $T$ **do**
  **for** $j \leftarrow 1$ **to** $n$ **do**
    $\delta_t(j) \leftarrow \underset{1 \le i \le n}{\text{MAX}}[\delta_{t-1}(i) \cdot a_{ij}] \cdot b_j(\theta_t)$
    $\psi_t(j) \leftarrow \underset{1 \le i \le n}{\text{ARGMAX}}[\delta_{t-1}(i) \cdot a_{ij}]$
  **end**
**end**



# XYYXX

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

# XYYXX

/* Termination

$$p^* \leftarrow \underset{1 \leq i \leq n}{\text{MAX}}[\delta_T(i)]$$
$$q_T^* \leftarrow \underset{1 \leq i \leq n}{\text{ARGMAX}}[\delta_T(i)]$$

p* = 0.006234

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | 2 |

# XYYXX

p* = 0.006234

/* Backtracking

**for** $t \leftarrow T-1$ **to** 1 **do**

$\quad q_t^* \leftarrow \psi_{t+1}(q_{t+1}^*)$

**end**

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | 2 |

# XYYXX

$p^* = 0.006234$

```
/* Backtracking
for t ← T − 1 to 1 do
  | q*_t ← ψ_{t+1}(q*_{t+1})
end
```

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | |
| 2 | |
| 3 | |
| 4 | 2 |
| 5 | 2 |

# XYYXX

$p^* = 0.006234$

```
/* Backtracking
for t ← T − 1 to 1 do
  | q*_t ← ψ_{t+1}(q*_{t+1})
end
```

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | |
| 2 | |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |

# XYYXX

$p^* = 0.006234$

```
/* Backtracking
for t ← T − 1 to 1 do
  | q*_t ← ψ_{t+1}(q*_{t+1})
end
```

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |

# XYYXX

p* = 0.006234

/* Backtracking

**for** $t \leftarrow T-1$ **to** 1 **do**

$q_t^* \leftarrow \psi_{t+1}(q_{t+1}^*)$

**end**

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |

# XYYXX

$p^* = 0.006234$

| δ | 1 | 2 |
|---|---|---|
| t = 1 | 0.3 | 0.175 |
| 2 | 0.126 | 0.042 |
| 3 | 0.0529 | 0.0113 |
| 4 | 0.0148 | 0.0111 |
| 5 | 0.0041 | 0.0062 |

| ψ | 1 | 2 |
|---|---|---|
| t = 1 | 0 | 0 |
| 2 | 1 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 2 |

| | q* |
|---|---|
| t=1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |

θ: XYYXX

State sequence: 1 1 1 2 2

Probability of observing θ, given the state sequence: 0.0062234

$p^* = 0.006234$

| | $q^*$ |
|---|---|
| t=1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |